Expansion Coefficients Through Laser Interferometry

1. Introduction

The laser interferometer is based on the interferometer used by Michelson in 1887 to measure the speed of the earth through the "ether". Although this was a negative experiment, interferometry has since found an enormous number of applications and we will explore one of them in this experiment. The interferometer takes a single beam of light, splits it in two, allows each beam to travel a separate path and finally combines the two again into a single beam.

Consider the intensity of the beam leaving the half silvered mirror. The electric field here is the sum of that from both paths

\[ E = E_0 \cos \left( \frac{2\pi}{\lambda} l_1 + \omega t \right) + E_0 \cos \left( \frac{2\pi}{\lambda} l_2 + \omega t \right) \]

where \( \lambda \) is the wavelength of the light source. The intensity is then proportional to the time average of \( E^2 \) (where \( \langle \cdot \rangle_t \) denotes time average).

\[ I \propto \langle E^2 \rangle_t = E_0^2 \left\langle \cos \left( \frac{2\pi}{\lambda} l_1 + \omega t \right) + \cos \left( \frac{2\pi}{\lambda} l_2 + \omega t \right) \right\rangle_t \]

\[ = E_0^2 \left[ 1 + \cos \left( \frac{2\pi}{\lambda} \left( l_1 - l_2 \right) \right) \right] \]

We can see that relative changes in the path length will result in changes in the intensity from zero to twice that of the original beam (make sure you understand why this is so). As a result the interferometer can be used to measure changes in the optical path length of one of the arms, the other
remaining fixed. The change in path length, $\Delta l$, required to go from one intensity peak to another is given by $\Delta l = \lambda/2$

The interferometer is a versatile instrument, which can be used to measure many optical properties such as index of refraction, absorption, reflection, etc. It is also very commonly used to measure changes in length, as is clear from the term $(l_1 - l_2)$ in the expression that governs the interference fringes. Here we will use it to measure a change in distance in one of the interferometer arms. One of the mirrors will be "attached" to the end of a bar of metal. This bar will be heated and then allowed to cool. As the bar cools it will contract causing the mirror to move and thereby fringes to change. This will allow us to measure the thermal expansion of the material in question, as well as the cooling rate.

**Apparatus:**
The following schematic illustrates the apparatus to be used.

![Schematic diagram of interferometer](image)

A copper-constantan thermocouple is used to measure the temperature of the metal bar and the thermal emf is measured by a digital meter.

The photo-resistor that detects changes to the interference pattern is also connected to the same digital meter. This meter is able to switch between a number of different inputs and can operate under computer control.

**Prelab Questions:**

1. Tables are available to calibrate the copper-constantan thermocouple used to measure temperature in this experiment (tables are in the lab or found in several handbooks, including chemistry's CRC Handbook or there is one at the end of the convection lab). Refer to one of these tables and use the data to fit to a polynomial in the temperature range of your experiment (room temperature to about 50 C). This fit to the calibration table will be useful in the performance of your experiment and the subsequent analysis.

2. The thermal expansion coefficient of copper near room temperature is $16.7 \times 10^{-6} \text{ K}^{-1}$. You will be measuring this thermal expansion coefficient using a bar of copper (15 cm long) that is mounted into one arm of an interferometer. Estimate how many fringes you will see in the interferometer as the bar cools from 50 C to room temperature.
**Procedure:**

Set up the interferometer as outlined in the appendix so that the photo-resistor is at the center of a clear circular interference pattern; this might take a little time and patience! Note that the wavelength for the He-Ne laser is 632.8 nm.

With the cylindrical copper bar in place, use the heat gun to warm the bar (and nothing else!) to 45-50°C. The object is to get a graph of the fringe count against temperature as the bar cools freely for 20-30 minutes. Do a trial run or two to establish an appropriate procedure. There is a program available on the computer that automates the collection of the temperature and photodetector output measurements.

Try not to disturb the apparatus while the experiment is under way, vibrations and bumps will adversely affect your interference pattern.

Double-click on the program icon on the computer's desktop to start it. Place the mouse on top of each of the icons to see what they do. Practice a few times to make sure you understand how the program works. You should probably leave the meter in 'strip-chart' mode so that you can see the progress of the experiment as it happens. The default values for the data rate and total number of points are probably fine, although you should check this on your trial run. Before each run, you should press the 'reset to defaults' button, followed by 'init' before starting the scan. You can monitor the measurements from each voltage source by rotating the knob on the panel of the HP34970A meter. Channel 102 is the photodetector and channel 103 is the thermocouple.

After you have collected the data you want, you can stop the run and save the data. When you select Save from the File menu, the data will appear in an editor window from which you can actually save it. The data has four columns, two of them are the time (in s) at which the associated measurement was made.

You will need a USB flash drive to retrieve the data from the computer.

In the data analysis, the goal is to come up with an accurate way to “count” the fringes and thus determine the expansion coefficient \( \alpha = (1/L)(dL/dT) \) and the cooling rate of the bar.

Repeat the procedure for the rectangular invar bar.

For nickel-steel alloys the linear expansion coefficient is a function of the percentage of nickel; the alloy for 36% nickel is called invar and for this optimum proportion the alloy has minimum expansion coefficient. Because of its exceptionally low expansion coefficient, invar is used in situations requiring low thermal strains. In the experiment there is the possibility that warming of the base results in excess or shortfall of the observed expansion compared with the true expansion ... which is more likely? Now, provided that the experimental situations are similar for the two cases you can correct this systematic error by using the 'book'-value for invar to estimate a correction. Does such a correction bring your observed value for copper closer to the tabulated values?

Tabulated linear expansion coefficients:

<table>
<thead>
<tr>
<th>Material</th>
<th>(10^{-6} \text{ K}^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>copper</td>
<td>16.7</td>
</tr>
<tr>
<td>invar</td>
<td>0.9                        (1)</td>
</tr>
<tr>
<td></td>
<td>0.8                        (2)</td>
</tr>
</tbody>
</table>

1) Physical and Chemical Constants, Kaye and Laby, 1948.
The lens (beam-spreader) in its black square housing should not be placed over the laser exit aperture until step 5 below.

1. Start by covering the input to the photodetector with a piece of paper and then removing the beam-spreader from the front of the laser.
2. Get mirror A aligned by aiming the directly reflected beam (path abcda) so that it goes back on itself. There are three screws on the back of the mirror that can be used to adjust its orientation. Don't overtighten the screws! If you loosen them enough, they will fall out. To identify the directly reflected beam, you can block mirror B. The spot you want should then be the only spot that doesn't move when you rotate the beam splitter. This is difficult to align perfectly, but try to get the reflected spot as close to the outgoing beam from the laser as you can.
3. Adjust the screws on mirror B so that they are all evenly loose, so that mirror B should be close to aligned. Now adjust the orientation of the beam splitter until the two spots (cdf and edf) are aligned horizontally.
4. Adjust mirror B (and the beam splitter rotation if necessary) until the two spots overlap.
5. When the two spots at the detector converge at the same point 'f' it should be easy to produce the circular interference pattern. Place the lens over the laser aperture and move it around a bit until the center of the pattern is at the same position as the two spots you just aligned. There should be a nice pattern of circular fringer. Move the detector to center position, or first (if necessary) fine-tune the mirror orientations until the circular fringes are centred on a more convenient position.