Measurement of the Cosmic Ray Flux

Introduction:

The surface of the earth is constantly being bombarded by a stream of subatomic particles known as cosmic rays. These rays are mostly muons, and their flux and energy are high enough to contribute a significant amount to our average dose of ionizing radiation - about 0.4 mSv (40 mrem). The purpose of this experiment is to measure their intensity and angular distribution.

Terrestrial cosmic rays are formed when high energy primary cosmic rays, mostly protons of uncertain origin, strike the top of the earth's atmosphere. They interact with nuclei and produce pions via the strong interaction. The pions come in three charge states: \( \pi^+ \), \( \pi^0 \), and \( \pi^- \). The charged pions decay with a mean life of 26 ns via the weak interaction into a muon and a neutrino. The neutral pions decay very rapidly via the electromagnetic interaction into two gamma rays. The interactions can be represented thus:

\[
\begin{align*}
pp & \rightarrow pn\pi^+ \text{ or } pp\pi^0 \\
\pi^+ & \rightarrow \mu^+ \nu \\
\pi^- & \rightarrow \mu^- \nu \quad (26 \text{ ns}) \\
\pi^0 & \rightarrow \gamma \gamma \quad (10^{-16} \text{ s})
\end{align*}
\]

Of these reaction products, only the muons and the neutrinos make it to the ground. The protons, neutrons and gamma rays interact too frequently to get very far. The neutrinos interact so rarely that they could make it through a light year of water. They are thus extremely difficult to detect (the Sudbury neutrino observatory used by Chris Waltham of our department detects solar neutrinos with a gigantic sphere of heavy water deep inside a nickel mine). It is the muons which will concern us here.

Muons are very important particles: they are heavy counterparts of electrons (and positrons) in the so-called "Second Generation" of particles. The "First Generation" consists of the electron, its neutrino, and the quarks which make up protons and neutrons ("up" and "down"). Muons were first discovered in cosmic ray tracks in the 1930's.

The muons are highly relativistic; their mean energy is 2 GeV (2 billion electron volts), compared to a mass of 106 MeV/c^2. Their mean life at rest is 2.2 \( \mu \)s, which, when increased by time dilation, is ample enough to get them down to earth. They are charged, which means they leave a trail of ionization in matter; this makes them straightforward to detect.

Apparatus:

We will detect the muons with a pair of thin plastic scintillators. Scintillators are materials which convert the trail of ionization left by a charged particle into a small flash of light: about 2000 photons per mm of track. Even considering inefficiencies in light transport and detection, this is more than enough to be "seen" by a high-gain detector like a photomultiplier tube (PMT).
Two scintillators will be arranged as a crude "telescope" (figure 1). A cosmic muon will fire both in rapid succession; a muon which has made it through an air density x thickness of 1 kg/cm$^2$ will have no trouble passing 0.3 g/cm$^2$ of plastic. For an "event" we will demand a COINCIDENCE of hits in the scintillators. This will roughly define the muon's direction and also discriminate against background noise in individual detectors and other short range radioactivities.

Before we proceed, a note on what we CANNOT measure with this apparatus.

We cannot determine the muon's energy: for highly relativistic particles the ionization trail is almost independent of energy, so the size of each signal is only a function of the path length in the scintillator. In addition, every particle is essentially travelling with the speed of light, so timing between the two scintillators is no help.

We cannot measure the charge state (+ or -) of the muon: the ionization trail is independent of the sign of the charge on the primary particle. The muon lifetime experiment in the 409 lab has some sensitivity to the ratio of $\mu^+$ to $\mu^-$ (about 1.3; crudely, there are more ways to make a $\mu^+$ than a $\mu^-$).

A note on the logic modules

The language high energy physicists use to talk about logic is a little different from that found in standard Boolean algebra that you have encountered in your introduction to digital electronics and logic. Our first task is to convert the analog signal from the PMT into a logic signal, providing it is of sufficient size not to be noise. This is performed with a DISCRIMINATOR: the input is from the photomultiplier tube (PMT), and the output is -0.8 Volts (convention for logical TRUE) if the input is large enough, and 0 V (logical FALSE) if not. The duration of these output signals can be varied.

Two of these logic signals can be brought together at a COINCIDENCE unit. If they overlap in time at this unit a further logical output will be produced which is essentially the logic AND of the two inputs. In our case the lower detector is expected to fire a little after the upper as the muon passes...
between them, so the latter signal can be delayed appropriately by the addition of extra cable, premeasured lengths of which are found in the DELAY unit. The output of the coincidence unit, which registers passing muons, can be counted using a COUNTER or SCALER. The time period over which we wish to count muons is set using the TIMER, which turns the scaler on and off. The entire setup is shown below in figure 2.

![Figure 2: Schematic of the electronics used to detect the muons.](image)

**Prelab Questions:**

1. Consider the 2.0 GeV muons being studied in this experiment. In the laboratory frame of reference, how long do the muons take to travel from the upper atmosphere to the surface of the earth? How long do they take to travel between the two detectors in the telescope used in this experiment. What are these travel times in the muon’s reference frame?

2. Carefully read the rest of the experimental procedure and consider Appendix 1 on random coincidences. Make an estimate of the appearance of the “delay curve” that you will measure in part 6 of the procedure.

3. Can you imagine what the source of the angular dependence in the muon count rate might be? Try to come up with a hypothesis and estimate the angular dependence.
**Procedure:**

1. Set the high voltage on the PMT bases to 2 kV (this voltage level has been chosen so that each of the PMTs functions optimally). Start with the telescope pointing straight up. Observe the signals from the PMTs using the oscilloscope and draw a picture. There is random electronic noise as well as noise from external photons getting into the scintillators. Roughly estimate the height of the signal peaks.

2. Feed the PMT signal to one of the top pair of discriminators to convert it to a logic signal. Take the output from one of the top four 'OUT' terminals to the oscilloscope. With the screwdriver provided, set the width of the output pulse to 20 ns using the WIDTH adjustment.

3. Now disconnect from the oscilloscope and take the output of the discriminator to one of the counter inputs. Select the appropriate channel with DISPLAY SELECT. The counter is counting when the red LED under GATE is showing. For now set the TIME BASE (i.e gate-control) to EXT and operate using the COUNT/STOP/RESET buttons. Later you will need to count for fixed periods set in units of minutes (1M) by the M and N settings (e.g. M=2 and N=1 will open the gate for Mx10^N = 20 minutes). With the screwdriver, adjust the discriminator threshold 'THR' until the pulse rate is very roughly in the 100 Hz region. Almost all of these pulses are spurious random counts. Note that only one of the counters will now achieve this pulse rate; for the other one adjust the threshold until the plateau is reached.

4. Connect the second PMT to the second of the top pair of discriminators, and repeat steps 1-3 for the second PMT.

5. Feed the output from the discriminator of the *top* PMT into one of the delay boxes, taking its output to either the A or B inputs of one of the coincidence units. (The inputs to the coincidence units are selected by pushing in the adjacent white buttons). Feed the output from the discriminator of the *bottom* PMT into the other input of the same coincidence unit. The delay box inserts an adjustable time delay into the signal from the top PMT so that the signals from the two PMTs can be arranged to arrive simultaneously at the coincidence unit. The time delays are given in nanoseconds by the sum from the selected switches plus an additional 2.5 ns - but the latter can be ignored in practice for there are other time delays in the system (roughly 3.3 ns per metre of coaxial cable).

6. Connect the output of the coincidence unit to the counter. Construct a delay curve - that is, a plot of count rate (counts per minute) against indicated delay time. You are now counting coincidences so the count rate is much reduced from before; you will need to count in the range 1-5 minutes. Take counts, increasing the indicated delay time in 1 or 2 ns steps from zero. Take counts until the rate drops and then levels out. In your lab report explain
   i. the shape of the delay curve;
   ii. your choice of an appropriate delay time that you will now fix for the remainder of the experiment.

7. Measure the coincidence count rate for 15° intervals in θ, starting in the vertical position. Use a protractor to measure the angle. To get adequate statistics 5-10 minute counts will be needed. The error in the count is \( \sqrt{n} \), so 150 counts in 10 minutes gives an error of ±12; this yields a rate of 15.0±1.2 counts per minute. Take at least one measurement at 90°; you may have to count for 30 minutes to get enough events.

8. Estimate the number of counts per minute that might be expected on the basis of random coincidence only; this is explained in Appendix I. You will need to measure the total count rate (background plus true events) for each tube separately, as observed in the initial
setting-up. Explain the connection between the random coincidence rate and your delay-curve of part 6.

9. Convert the muon count rates to the units for muon flux, \( \text{sr}^{-1} \text{m}^{-2} \) (counts per second, per steradian of the angle of view, per unit area of the lower detector). An approximate theory to do this conversion is given in Appendix II. The separation between the scintillation detector paddles is 0.30m, and the smallest is circular with diameter 0.20m.

10. Plot the observed muon flux as a function of angle \( \varepsilon \). Include error bars for both axes. Explain the general shape of the curve.
APPENDIX I: Random Coincidences


The figure shows pulses from two channels. The delay on channel 2 has been compensated by a shift of time origin so that coincidences do appear as simultaneous pulses on the diagram.

Besides true coincidences of muon events, each channel records random pulses arising from circuit noise and from muons that are detected by one channel but do not pass through the scintillator of the other channel. Occasionally the random pulses overlap, giving a spurious 'event' called a random coincidence. The diagram shows random pulses appearing on both channels, including two random coincidences - one example for the situation when the coincidence count is 'set' by a channel 1 pulse and then is subsequently triggered by a channel 2 pulse, and one example for the reverse situation.

Suppose that the true coincidence rate is $N_C$. Let the pulse rates be $N_1$ and $N_2$ for channels 1 and 2 respectively, and let the corresponding pulse widths be $\tau_1$ and $\tau_2$.

On channel 1 the rate for random (non-event) pulses is $N_1 - N_C$, so channel 1 is 'live' for counting random coincidences for $(N_1 - N_C)\tau_1$ seconds of each second of running time. For channel 2, the rate for random pulses is $N_2 - N_C$. Hence the random coincidence rate for a pulse on 1 followed by a pulse on 2 is

$$N_{12} = (N_1 - N_C)\tau_1(N_2 - N_C)$$

Similarly, the random coincidence rate for a pulse on 2 followed by a pulse on 1 is

$$N_{21} = (N_2 - N_C)\tau_2(N_1 - N_C)$$

The total random coincidence rate is the sum of these

$$N_R = (N_1 - N_C)(N_2 - N_C)(\tau_2 + \tau_2)$$

In practice both $N_1$ and $N_2$ are much greater than $N_C$ so it is permissible to simplify the expression to

$$N_R = N_1 N_2 (\tau_2 + \tau_2)$$
APPENDIX II: Telescope Geometry

The solid angle of acceptance for muons arriving at the element of area $\delta A_2$ is

$$\Delta \Omega_2 = A_1/d^2 \approx A_1/d^2$$

where $A_1$ is the area of the spherical cap that is centered on $\delta A_2$ and bounded by the perimeter of $A_1$, the area of the top detector. The solid angle 'seen' by other off-axis area elements of $A_2$ is slightly different, but the error is not too large if $\Delta \Omega$ is small. A rough estimate of the error range can be made for the geometry of the experiment.

Suppose that the muons arriving from different directions (but within the admittance angle of the telescope) have a flux of $N$ muons per second per steradian of solid angle per m$^2$ of detector $A_2$. Then the detection rate for element $\delta A_2$ is

$$\delta N_C = N \delta A_2 \Delta \Omega_2 \approx N \delta A_2 A_1/d^2$$

Summing over the total area $A_2$, the total count rate is

$$N_C = N A_1 A_2/d^2$$

This relationship is used to determine the muon flux $N$ from the observed coincidence rate $N_C$. In the binder at the back of the bench in the lab there is a page that shows how to perform a more precise calculation by numerical methods, but with the approximations used there the final result is identical to the one obtained here.