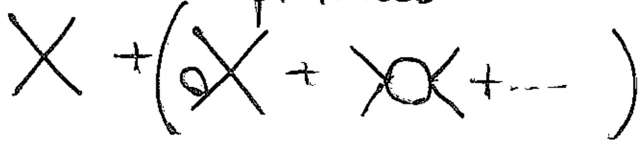
LAST TIME: UV divergences in higher-order amplitudes



solution: include cutoff

$$S[m,\lambda) \to S[m,\lambda,\Lambda]$$

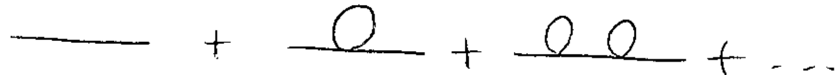
$$\phi(x) \to \phi_\Lambda(x) = \int_{|\vec{p}|<\Lambda} d^3\vec{p}\; e^{i\vec{p}\cdot\vec{x}}\,\phi(\vec{p})$$

can define physical parameters $m_{PHYS}(m,\lambda,\Lambda)$, $\lambda_{PHYS}(m,\lambda,\Lambda)$ so that any low-energy observable

$$\sigma(m,\lambda,\Lambda) = \sigma(m_{PHYS},\lambda_{PHYS}) + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$

can take $\Lambda \to \infty$ to get finite result.

m_{PHYS} = physical mass
= pole in full propagator.

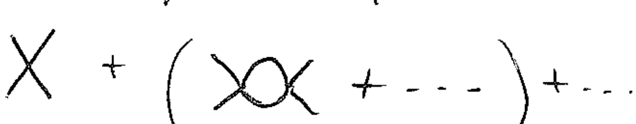


leading order $\quad \dfrac{1}{p^2-m^2} + \ldots$

pole at $p^2 = m^2$

full result: pole at $p^2 = m_{PHYS}^2 = m^2 + \lambda\cdot\Lambda^2 + \ldots$

λ_{PHYS}: full scattering amplitude at $p \to 0$



$$\lambda_{PHYS} = \lambda + \lambda^2 \ln(\Lambda) + \ldots$$

MORE GENERAL:

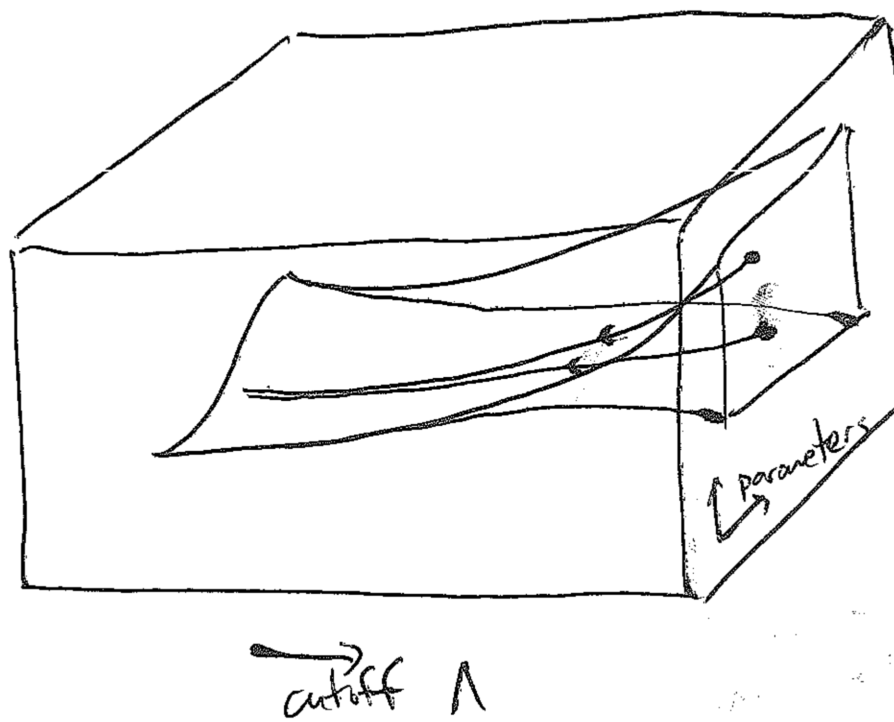
- Can start with

$$S[m_i, \lambda_i, \phi_\Lambda]$$

↑ all possible couplings

- low-energy physics $E \ll \Lambda$ depends only on small number of physical parameters

$$\sigma(\underbrace{m_i, \lambda_i}_{\text{large \# parameters}}, \Lambda) = \sigma(\underbrace{m^a_{PHYS}, \lambda^a_{PHYS}}_{\text{small \# parameters.}}) + \mathcal{O}\left(\frac{1}{\Lambda}\right)$$



- many UV theories have same IR physics.

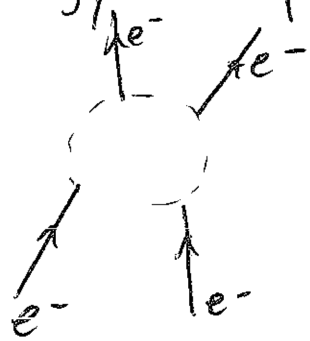
- can find simple theory with cutoff $\Lambda \gg E$ that has same low E predictions as real theory.

IR DIVERGENCES:

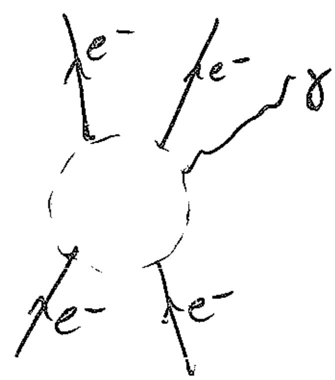
$\int d^4k \cdot$ divergent even with cutoff $|k| < \Lambda$

→ These cancel when we include all diagrams relevant to a physical measurement.

Real experiment: particle detectors have lower limit on energy of photons detected



indistinguishable from

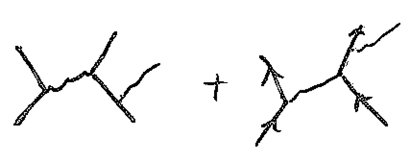


if $E_{photon} < E_{min}$

Really want cross section for

$$e^-e^- \to e^-e^- + \text{arbitrary number of "soft" photons } E < E_{min}$$

$$\sigma = \sigma(e^-e^- \to e^-e^-) + \int_0^{E_{min}} dE\, \sigma(e^-e^- \to e^-e^- \gamma(E))$$



↖ divergences cancel. ↗

FULL ANSWER

Full answer for e^- scattering off of very heavy charged particle:

$$\left(\frac{d\sigma}{d\Omega}\right)_{meas} = \left(\frac{d\sigma}{d\Omega}\right)_0 \times \exp\left(-\frac{\alpha}{\pi} f(\vec{p}-\vec{q})^2 \ln\left(\frac{(\vec{p}-\vec{q})^2}{E_{min}}\right)\right)$$

$\longrightarrow 0$ when $E_{min} \to 0$

cross section with no photons $= 0$

ACCELERATED CHARGES RADIATE!

radiated photons = BREHMSSTRAHLUNG.