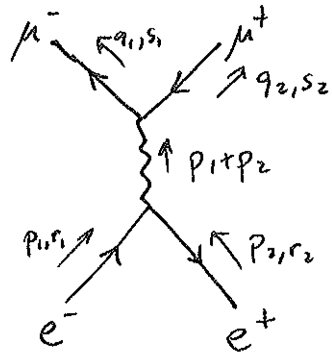
LAST TIME: cross section for $e^+e^- \to \mu^+\mu^-$



$$iM = \frac{ie^2}{(p_1+p_2)^2}\,\bar{u}^{s_1}(q_1)\gamma^\mu v^{s_2}(q_2)\,\bar{v}^{r_2}(p_2)\gamma_\mu u^{r_1}(p_1)$$

for cross section need $|M|^2 = MM^*$

AVERAGE over initial spins (if don't know)

SUM over final spins (if don't measure)

NEED: $(\bar{\psi}\gamma^\mu\chi)^* = \chi^\dagger\gamma^{\mu\dagger}\gamma^{0\dagger}\psi$

$$= \chi^\dagger\gamma^0\gamma^\mu\psi$$

$$\Rightarrow \boxed{(\bar{\psi}\gamma^\mu\chi)^* = \bar{\chi}\gamma^\mu\psi}$$

Want: $\frac{1}{2}\sum_{r_1}\frac{1}{2}\sum_{r_2}\sum_{s_1,s_2}|M_{r_1r_2s_1s_2}|^2$

$$= \frac{1}{4}\sum_{\substack{r_1,r_2\\ s_1,s_2}} \bar{u}^{s_1}(q_1)\gamma^\mu v^{s_2}(q_2)\bar{v}^{s_2}(q_2)\gamma^\nu u^{s_1}(q_1)$$
$$\bar{v}^{r_2}(p_2)\gamma_\mu u^{r_1}(p_1)\;\bar{u}^{r_1}(p_1)\gamma_\nu v^{r_2}(p_2)$$

spin sums: use:
$$\boxed{\begin{aligned}\sum_s u^s_\alpha(p)\bar{u}^s_\beta(p) &= (\not{p}+m)_{\alpha\beta}\\ \sum_s v^s_\alpha(p)\bar{v}^s_\beta(p) &= (\not{p}-m)_{\alpha\beta}\end{aligned}}$$

Get e.g. $\bar{u}^{s_1}_\alpha(q_1)\gamma^\mu_{\alpha\beta}v^{s_2}_\beta(q_2)\bar{v}^{s_2}_\rho(q_2)\gamma^\nu_{\rho\sigma}u^{s_1}_\sigma(q_1)$

$$\to \gamma^\mu_{\alpha\beta}(\not{q}_2 - m_\mu)_{\beta\rho}\gamma^\nu_{\rho\sigma}(\not{q}_1 + m_\mu)_{\sigma\alpha}$$

$$= \mathrm{tr}\left(\gamma^\mu(\not{q}_2 - m)\gamma^\nu(\not{q}_1 + m)\right)$$

$$= 4\left(q_1^\mu q_2^\nu + q_1^\nu q_2^\mu - q_1\cdot q_2\,\eta^{\mu\nu} - m_\mu^2\eta^{\mu\nu}\right)$$

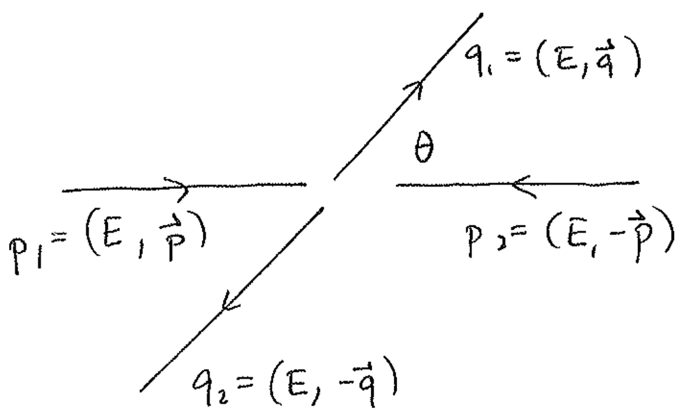
Need:
$$\boxed{\begin{aligned}&\mathrm{tr}(\gamma^\mu)=0 \qquad \mathrm{tr}(\gamma^\mu\gamma^\nu)=4\eta^{\mu\nu} \qquad \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\lambda)=0\\ &\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma) = 4(\eta^{\mu\nu}\eta^{\lambda\sigma} + \eta^{\nu\lambda}\eta^{\sigma\mu} - \eta^{\mu\lambda}\eta^{\nu\sigma})\end{aligned}}$$

$$\frac{1}{4}\sum|M|^2 = \frac{8e^4}{(p_1+p_2)^4}\left[(p_1\cdot q_1)(p_2\cdot q_2)+(p_1\cdot q_2)(p_2\cdot q_1)+m_\mu^2\,(p_1\cdot p_2)+m_e^2\,(q_1\cdot q_2)\right]$$

choose frame: C.O.M. simplest

must have $E_e \gg m_e$ so approximate $m_e = 0$

$\therefore E \approx |\vec{p}_1| = |\vec{p}_2|$



$$\frac{1}{4}\sum|M|^2 = \frac{8e^4}{(2E)^4}\Big((E^2 - E|\vec{q}|\cos\theta)^2 + (E^2 + E|\vec{q}|\cos\theta)^2 + 2m_\mu^2 E^2\Big)$$

$$= e^4\left[\left(1+\frac{m_\mu^2}{E^2}\right)+\left(1-\frac{m_\mu^2}{E^2}\cos^2\theta\right)\right]$$

Plug into cross section formula, integrate over all energies/momenta that are fixed by δ fns:

$$d\sigma = \frac{1}{2E_{p_1}\cdot 2E_{p_2}\cdot|v_1-v_2|}\left(\frac{1}{4}\sum|M|^2\right)\underbrace{\frac{d^3q_1}{(2\pi)^3 2E_{q_1}}\frac{d^3q_2}{(2\pi)^3 2E_{q_2}}(2\pi)^4\,\delta^4(p_1+p_2-q_1-q_2)}_{\text{same as for } \phi\phi\to\phi\phi}$$

gives $\dfrac{|q_1|}{(2\pi)^2\cdot 4\cdot E_{cm}}\cdot d\Omega$

$\alpha = \frac{1}{137}$

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4\pi}\right)^2\frac{1}{16E^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left[\left(1+\frac{m_\mu^2}{E^2}\right)+\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta\right]$$

$$\boxed{\sigma_{TOT} = \int_{\text{sphere}}\frac{d\sigma}{d\Omega}\cdot d\Omega = \frac{4\pi\alpha^2}{3E_{cm}^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left(1+\frac{1}{2}\frac{m_\mu^2}{E^2}\right)}$$

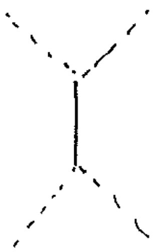
→ excellent agreement w. experiment

→ can be used to measure mass of particle.

~~Q: How many μ produced Bec 2 in beam~~

very large E: $\sigma \sim \dfrac{1}{E^2}$ (dimensional analysis) if m_e, m_μ irrelevant.

Interactions via massive particle



e.g. HW

propagator $\frac{1}{(p_1+p_2)} \longrightarrow \frac{1}{(p_1+p_2)^2 - M^2}$

final result has factor $\frac{1}{(E_{cm}^2 - M^2)^2}$

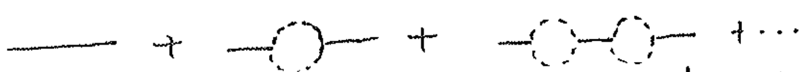
① $E_{cm} \ll M$ propagator acts like const $-\frac{1}{M^2}$

physics same if we replace $\mu \phi^2 \Phi$ with $\frac{\mu^2 \phi^4}{M^2}$ 

② $E_{cm} \gg M$ mass unimportant

③ $E_{cm} = M$ result blows up

Higher orders smooths this out since particle unstable.



shifts $M \to M - \frac{i\Gamma}{2}$ in propagator

decay rate

e^{-iMt} evolution $\to e^{-iMt} e^{-\frac{\Gamma t}{2}}$

decay to other particles

$$\sigma \sim \frac{1}{(E-M)^2 + \frac{\Gamma^2}{4}} \quad \text{if} \quad \Gamma \ll M$$

"Resonance" : signature of new unstable particle

Energy $\to$ particle mass

Width $\to \frac{1}{\text{lifetime}}$.