SO FAR: to compute decay rates, cross sections

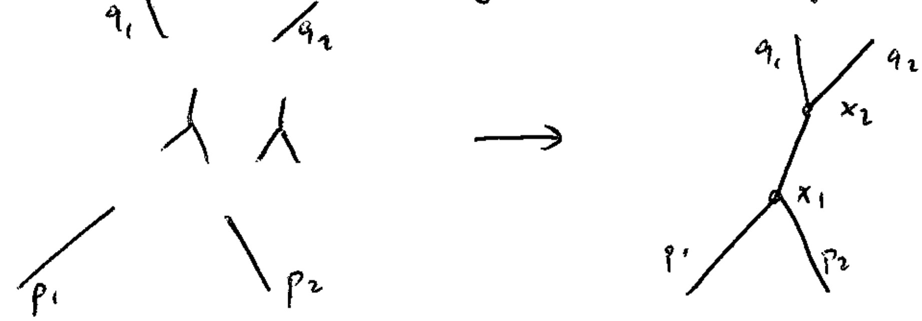
compute $\mathcal{M}_{fi}$ $\longrightarrow$ plug in to $d\sigma$, $d\Gamma$ formula

$\downarrow$

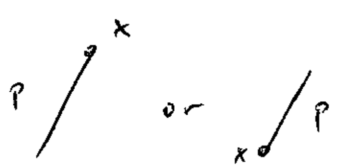
integrate over quantities fixed by δ fns, other quantities that you want to include

Feynman Diagrams: quick way to compute $\mathcal{M}_{fi}$

$$\langle 0| a_{q_1}\sqrt{2\omega_{q_1}} \cdots a_{q_m}\sqrt{2\omega_{q_m}} \int d^4x_1\, \phi_i(x_1) \cdots \phi_i(x_1) \int d^4x_2 \cdots \sqrt{2\omega_{p_1}}\, a^\dagger_{p_1} \cdots a^\dagger_{p_n}|0\rangle$$



① x - integrals

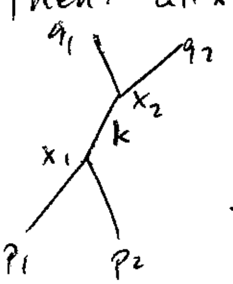


$\sqrt{2\omega}\langle a\,\phi_i(x)\rangle$ $\quad$ $\sqrt{2\omega}\langle\phi_i(x)\, a^\dagger\rangle$ $\quad$ gives $e^{\pm ip\cdot x}\cdot\begin{cases}1 & \text{scalar}\\ u \text{ or } v & \text{spinor}\\ \varepsilon^\mu & \text{vector}\end{cases}$

gives $D_F(x_1-x_2)$

write as $\displaystyle\int\frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x_1-x_2)}\, D_F(k)$

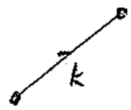
Then: all x-integrals give δ fns in momentum



$$\int d^4x_1 \int d^4x_2\, e^{-ip_1\cdot x_1} e^{-ip_2\cdot x_2} \cdot \int\frac{d^4k}{(2\pi)^4}\, e^{-ik(x_2-x_1)} D_F(k)\, e^{iq_1\cdot x_2} e^{iq_2\cdot x_2}$$

$$\rightarrow \int\frac{d^4k}{(2\pi)^4}\,(2\pi)^4\delta(p_1+p_2-k)\,(2\pi)^4\delta(k-q_1-q_2)\,D_F(k) \rightarrow \underbrace{(2\pi)^4\delta(p_1+p_2-q_1-q_2)}_{\text{NOT PART OF } \mathcal{M}_{fi}} D_F(p_1+p_2)$$

Summary: Just assign momenta to internal lines consistent w. energy / mom. conservation

for  get factor of

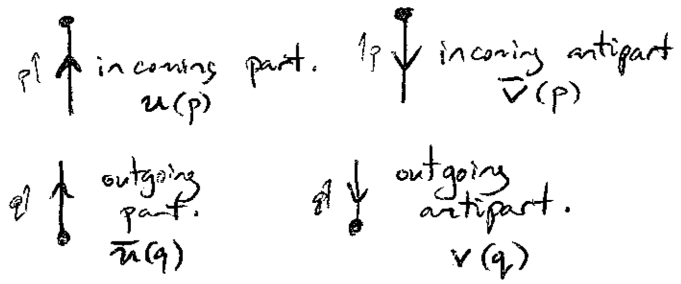
$$D_F(k) = \begin{cases} \dfrac{i}{k^2-m^2+i\varepsilon} & \text{scalar} \\ \dfrac{i(\not{k}+m)_{\alpha\beta}}{k^2-m^2+i\varepsilon} & \text{spinor} \\ \dfrac{-i\eta_{\mu\nu}}{(k^2+i\varepsilon)} & \text{massless vector} \end{cases}$$

(see notes)

② particles vs antiparticles

$$\langle \psi\,\psi \rangle = \langle \bar\psi\,\bar\psi \rangle = 0$$
$$\langle a\psi \rangle = \langle \bar\psi\, b^\dagger \rangle$$
$$= \langle \bar\psi\, a^\dagger \rangle = \langle b\bar\psi \rangle = 0$$

∴ need notation for difference between particles & antiparticles



interaction $-q\bar\psi\gamma^\mu A_\mu\psi$  gives $iq\gamma^\mu_{\alpha\beta}$

Rules: - Draw diagrams
- Assign internal momenta.

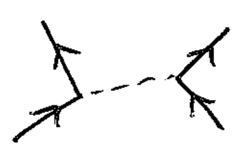
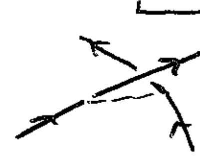
For each, $i\mathcal{M}$ = product of

N : # ways

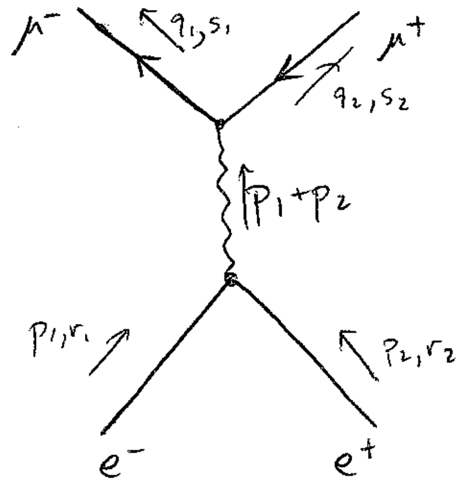
$\frac{1}{n!}$ if an interaction vertex appears n times.

appropriate factors for each vertex, incoming/outgoing/internal line.

for fermions $(-1)^{\#\text{crossings} + \#\bar\psi\psi \text{ contractions}}$ $\langle \bar\psi\psi \;\; \bar\psi\psi$

 vs  relative - sign.

EXAMPLE: Production of new particles in e^+e^- annihilations



$$i\mathcal{M} = \bar{u}^{s_1}(q_1)(-i\gamma^\mu e)v^{s_2}(q_2)$$
$$\bar{v}^{r_2}(p_2)(-i\gamma^\nu e)u^{r_1}(p_1)$$
$$\frac{-i\eta_{\mu\nu}}{(p_1+p_2)^2}$$

$$= \frac{ie^2}{(p_1+p_2)^2}\bar{u}^{s_1}(q_1)\gamma^\mu v^{s_2}(q_2)\bar{v}^{r_2}(p_2)\gamma_\mu u^{r_1}(p_1)$$

Cross section: need $|\mathcal{M}|^2$

Generally: have random assortment of initial particle spins, don't measure final particle spins

$\therefore$ Sum cross section (not amplitude) over final particle spins

Average over initial particle spins

Want $\frac{1}{2}\sum_{r_1}\frac{1}{2}\sum_{r_2}\sum_{s_1,s_2}|\mathcal{M}_{r_1 r_2 s_1 s_2}|^2$

Note: $$(\bar{\psi}\gamma^\mu\chi)^* = \chi^\dagger\gamma^{\mu\dagger}\gamma^{0\dagger}\psi$$
$$= \chi^\dagger\gamma^0\gamma^\mu\psi$$
$$= \bar{\chi}\gamma^\mu\psi$$