## A SCIENCE ONE PHYSICS TUTORIAL Work and the Dot Product

Since the beginning of the semester a friend of yours has been living with the worst roommate in the world. Your friend has finally managed to trade rooms with some poor soul in their building, and they've asked you to help them move. In return for helping them move they say they're going buy you pizza.


You want to be sure they buy enough pizza for you, so you decide to calculate how much energy you'll use moving the boxes. You remember from Science One that the amount of energy required to move an object given a force is

$$
\begin{equation*}
\Delta W=F_{x} \Delta x+F_{y} \Delta y+F_{z} \Delta z \tag{1}
\end{equation*}
$$

We can use this to define the dot product for vectors

$$
\Delta W=\vec{F} \cdot \Delta \vec{r}
$$

A slice of pizza has 200 Calories, and 1 Calorie is about 4200 Joules.

## Question 1

a) The simplest way to move the boxes is to crouch down and push them horizontally along the floor. Each box weighs about 100 kg and the coefficient of friction between cardboard and dormitory floor linoleum is $\mu=0.3$. Your friend first asks you to push the boxes to the end of the hall, which is 30 m away. You move the boxes at a constant speed as not to do much work. How much work do you do moving a box down the hall?
b) If the box is moving at a constant speed, where does the energy from all this hard work go?

## Question 2

a) After seeing you struggle to move one box, your friend gets you a rope to pull the boxes. The rope allows you to pull on the box at an angle of $\theta=30^{\circ}$. Excited to have the rope, you no longer pull the box with a constant velocity. You measure the tension in the rope with a spring scale, which reads 500 N . Use equation (1) to evaluate the work done to move the 100 kg box 30 meters using the rope.

b) The dot product can also be evaluated by using

$$
\Delta W=\vec{F} \cdot \Delta \vec{r}=F \Delta r \cos \theta
$$

where $\theta$ is the angle between the vectors $\vec{F}$ and $\Delta \vec{r}$. Use this to evaluate the work it takes to pull a single box to your friend's room.

## Question 3

a) Your friend now wants you to move a box up a short set of stairs. You must get the box from point $A$ to point $B$. The figure below indicates two possible paths. You can lift the box over your head first and move the box over, or you can move the box close along the ground (such that there's no friction). The horizontal distance between $A$ and $B$ is 4 m , the vertical distance is 2 m . Calculate the work done for each path.

b) Your friend now wants you to slide the box from one corner of the room to the other, as shown in the figure. The room is 4 m long and 2 m wide and the coefficient of friction is 0.50 and the boxes still have a mass of 100 kg . Calculate the work required to slide the box along each path.

c) What do you notice about the work done for each path in part a) and in part b). How are the forces different?
d) You now move the box along the path shown below. Write down an expression that evaluates the work done when the path is curved. What is the work done along this path?


## Question 4

Your friend has 20 boxes to move. If you add up all the steps to move a box ( $2 \mathrm{a}, 3 \mathrm{a}$ and 3 b ), how much pizza should your friend buy so you can replenish your energy?

## Question 5

Using $\vec{A} \cdot \vec{B}=A B \cos \theta$, show that $\vec{A} \cdot \vec{B}=A_{x} B_{x}+A_{y} B_{y}$, where $\vec{A}=A_{x} \hat{x}+A_{y} \hat{y}$ and $\vec{B}=B_{x} \hat{x}+B_{y} \hat{y}$. The dot product is distributive, just like regular multiplication.

