

Heat and Heat Capacity

Consider two bodies at different temperatures in thermal contact. Then internal or thermal energy from the hot body will flow to the cold body. The energy transferred is called *heat*. In thermodynamics heat always refers to an energy transfer. It is technically *incorrect* to say a body has a certain amount of heat.

One can increase the temperature of a body by increasing the internal energy through heat transfer (e.g. heating a liquid with a flame) or by doing work on the body (e.g. stirring a liquid, moving one surface past another friction, or bending a metal wire etc). For the moment we assume no work is being done and restrict the discussion to heat transfer. ($W=0$)

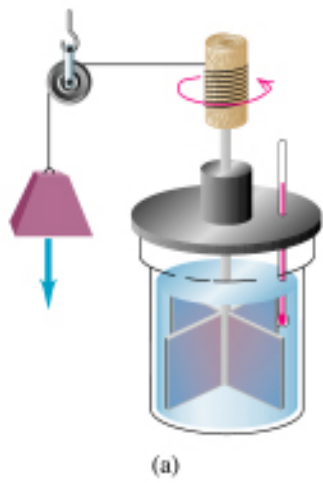
We use symbol Q for a finite amount of heat transfer whereas an infinitesimal amount of heat is dQ . The common units of Q are the cal (calorie) or Btu (British thermal unit)

- 1 cal is heat required to raise the temperature of 1gm of water from 14.5 °C to 15.5°C.
- kcal=1000cal is one food energy calorie
- 1Btu is the heat required to raise one pound water from 63°F to 64°F.

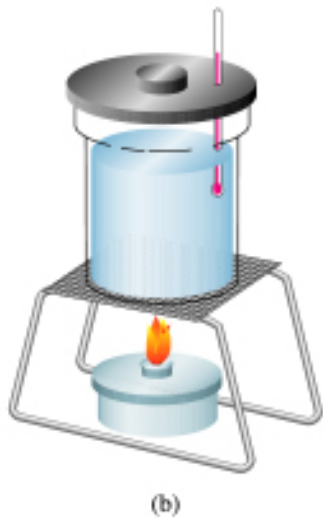
$$1\text{cal}=4.186\text{ J}$$

$$1\text{Btu}=778\text{ft.lb}= 1055\text{ J} = 252\text{ cal}$$

Change in Internal Energy of a body
equals heat transfer + work done
($\Delta U=Q+W$)



Work=Force x distance



Heat

Specific Heat Capacity

Consider a body of mass m at temperature T_0 . How much heat Q is required to raise the temperature by an infinitesimal amount dT ?

$$dQ = mc dT$$

dQ scales with the mass of an object since it takes twice as much energy change the temperature of body with twice the mass. The constant of proportionality c is called the **specific heat capacity**. c depends in detail on the material properties (i.e. constituent atoms, interactions between them, structure etc). In general it also depends on the temperature T_0 but for small finite changes in the temperature $\Delta T = T - T_0$ (away from any phase transition) it can be assumed to be a constant so that

$$Q = mc\Delta T$$

Specific heat of water at 15°C is 1 cal/gC° or 4190J/kg.K or 1 Btu/lb.F°. **See table 15-3 for a list of heat capacities.**

Example 15-8. Consider a 23 mg Si microchip absorbing 7.4 mW. What will T rise be after one minute if you don't design any cooling mechanism?

$$\Delta T = \frac{Q}{mc} = \frac{7.4 \times 10^{-3} \text{ J s}^{-1} \times 60 \text{ s}}{23 \times 10^{-6} \text{ kg} \times 705 \text{ J / kg} \cdot \text{K}} = 27 \text{ K}$$

Molar Heat Capacity (C)

Similarly one can define a molar heat capacity $C=Mc$ Where M is the mass per mole of a substance. Then using $m=nM$ where n is the number of moles

$$Q = nMc\Delta T = nC\Delta T$$

For water $C=Mc=(0.018\text{kg/mole}) 4190 \text{ J/kg.K}$
 $=75.4 \text{ J/mole.K}$

Heat capacities measured at constant pressure are denoted c_p or C_p . In gases heat capacities can also be measured at constant volume and are then denoted c_v or C_v . In general c_v and c_p are different.

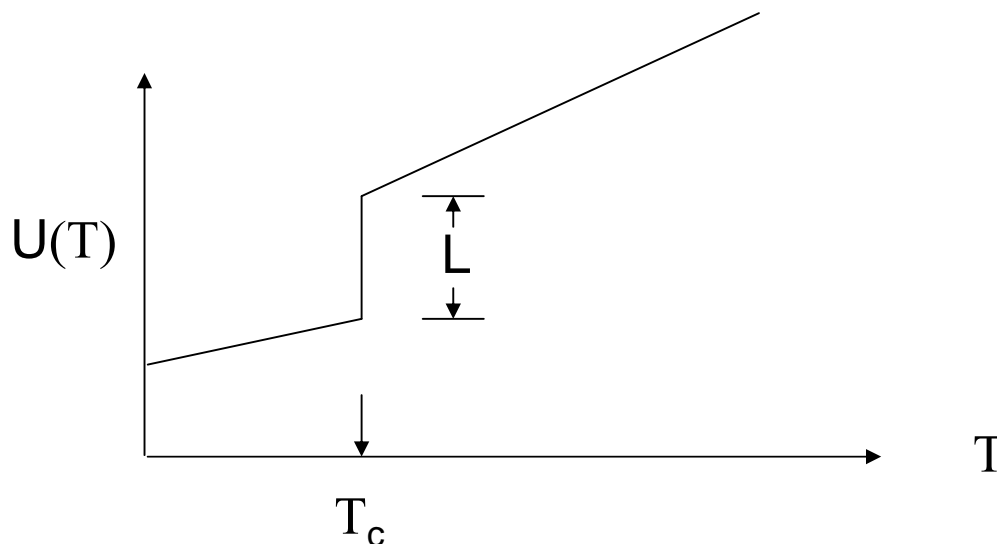
Note from table 15-3 heat capacities of all the elements are all about the same. Al ($M=27 \text{ g/mole}$) is 24.6 J/mole.K whereas Ag($M=108 \text{ g/mole}$) is 25.3 J/mole.K . *Thus the heat capacity depends on the number of atoms rather than their mass!*

This is the **rule of Delong and Petit** which we discuss later.

Latent Heat and Phase Transitions

As the temperature of substance passes through certain values (transition temperatures T_c) its physical properties may change suddenly as a result of a **phase transition**. It exists in one phase below T_c and another phase above T_c . e.g water vapor condenses into liquid water below 100°C and water freezes into ice at 0°C .

There are many different kinds of phase transitions in nature. The most well known type (call first order) involves a **latent heat**. This is the energy or heat per unit mass required to convert the substance from one phase to another at the transition temperature at T_c . Latent heat is given the symbol L .



Consider what happens when one supplies a steady flow of heat into a body. Below T_c temp rises steadily according to c_v of the lower phase. At T_c the temperature stops increasing, the two phases coexist the amount of the lower phase decreases while that of the upper phase increases.

Latent heats associated with vaporization and freezing

Latent heat associated with freezing is called the **heat of fusion** and denoted L_f . For water:

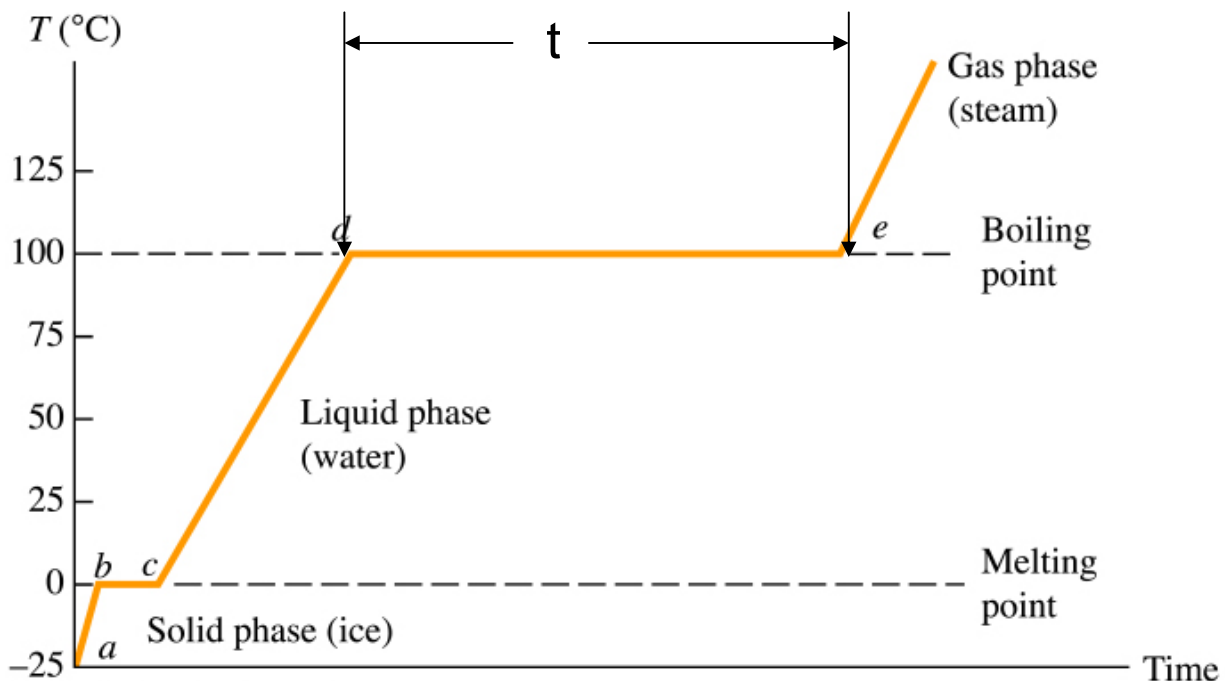
$$L_f = 3.34 \times 10^5 \text{ J/kg} = 79.6 \text{ cal/g}$$

Heat of vaporization is denoted L_v . For water

$$L_v = 2.256 \times 10^6 \text{ J/kg} = 539 \text{ cal/g.}$$

Consider a block of ice mass m being heated at a constant rate R . What is the temperature as function of time? At a phase transition the temperature stays constant for a time t during which the two phases coexist. All the energy goes into changing one phase to another.

$$t = mL_v / R$$



Example 1

Consider two bodies masses at different temperatures T_C and T_H . Bring them into thermal equilibrium. What is final temperature T given they have masses m_1 and m_2 and heat capacities c_C and c_H respectively. Assume no phase transition.

Energy conservation implies no change in the internal energy of the system (both bodies) after reaching

equilibrium i.e. $\Delta U_C + \Delta U_H = 0$:

$$\Delta U_C = Q_C = c_C m_C (T - T_C)$$

$$\Delta U_H = Q_H = c_H m_H (T - T_H)$$

thus

$$c_C m_H (T - T_C) + c_H m_H (T - T_H) = 0$$

Solving for T we obtain :

$$T = \frac{c_C m_C T_C + c_H m_H T_H}{c_C m_C + c_H m_H}$$

Exercise : Repeat this for three bodies, m_1 , m_2 and m_3 .

Example 2:

Now assume the colder body (C) is a solid (e.g. ice) at the melting temperature and that *all the ice melts* when put in thermal equilibrium with hot body (H). What is the final temperature given the latent heat of fusion is L_f ?

$$\Delta U_C = m_C L_f + m_C c_C (T - T_C)$$

$$\Delta U_H = m_H c_H (T - T_H)$$

As before since there is no change in the total internal energy of the system the sum of $\Delta U_C + \Delta U_H = 0$. Solving for T we find:

$$T = \frac{m_H c_H T_H + m_C c_C T_C - m_C L_f}{m_C c_C + m_H c_H}$$

Note this is similar to the previous example except the latent heat term lowers the average temperature.

Question: What is T if only a fraction (f) of the solid melts when the two bodies reach thermal equilibrium?

What are ΔU_C and ΔU_H ?

Question: What if you don't know if all the ice melts or not?

Latent heat cont'd Example 3

How much heat is required to bring 1500 g of ice at -50°C into steam?

$$\text{Step 1 } -50^{\circ}\text{C} \rightarrow 0^{\circ}\text{C}: Q_1 = mc_{\text{ice}} = 1.5\text{kg} \times 2\text{kJ/kg} \cdot \text{K} \times 50\text{K} = 150\text{kJ}$$

$$\text{Step 2 melt ice} \rightarrow Q_2 = mL_f = 1.5\text{kg} \times 334\text{kJ/kg} = 501\text{kJ}$$

$$\text{Step 3 } 0^{\circ}\text{C} \rightarrow 100^{\circ}\text{C}: Q_3 = mc_{\text{water}} = 1.5\text{kg} \times 4.186\text{kJ}/(\text{kg} \cdot \text{K}) \times 100\text{K} = 628\text{kJ}$$

$$\text{Step 4 boil water} \quad Q_4 = mL_v = 1.5\text{kg} \times 2256\text{kJ/kg} = 3384\text{kJ}$$

$$\text{Total} = 4663 \text{ kJ}$$

What is the temperature if you only supply 1000kJ?

$651\text{kJ} = (150\text{kJ} + 501\text{kJ})$ to just melt ice so 349 kJ left over.

$$349 \text{ kJ} = 1.5\text{kg} \times 4.186\text{kJ}/(\text{kg} \cdot \text{K}) \Delta T$$

$$\Delta T = 55.6\text{K}$$

$$T = 0^{\circ}\text{C} + 55.6^{\circ}\text{C} = 55.6^{\circ}\text{C}$$

Heat Transfer

Heat transfer from one body to another may take place in 3 distinct ways **conduction, convection and radiation.**

Conduction involves no change in atomic positions but energy of atomic vibrations (and free electrons) moves from hot regions to colder regions.

i.e. regions of high energy density to low energy. Consider a conducting rod of length L and area A connecting between two bodies at T_H and T_C .

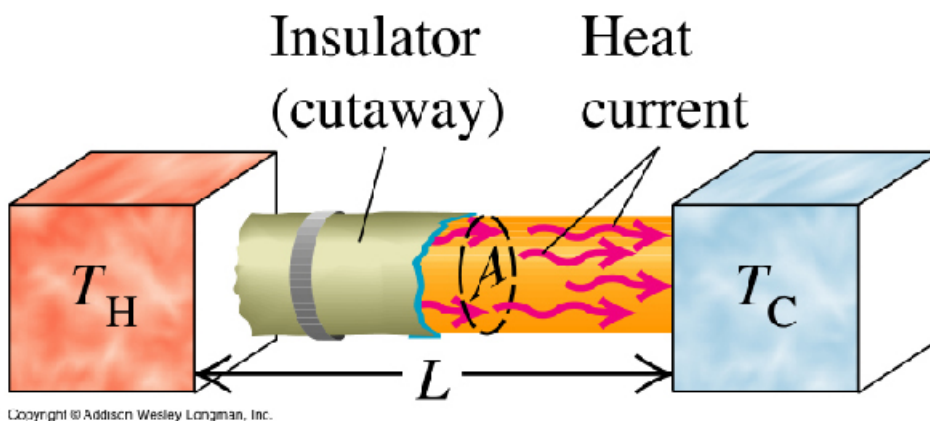


Fig. 15- 13

Then the rate of heat flow dQ/dt from hot to cold is called **heat current** or H and experimentally is given by:

$$H = kA \frac{T_H - T_C}{L}$$

where k is called the thermal conductivity and depends only on the properties of the material and not the dimensions. k has units $W/m.K$. A good conductor like Cu has $k=385$ $W/m.K$ whereas a good insulator like glass has $k=0.8$ $W/m.K$

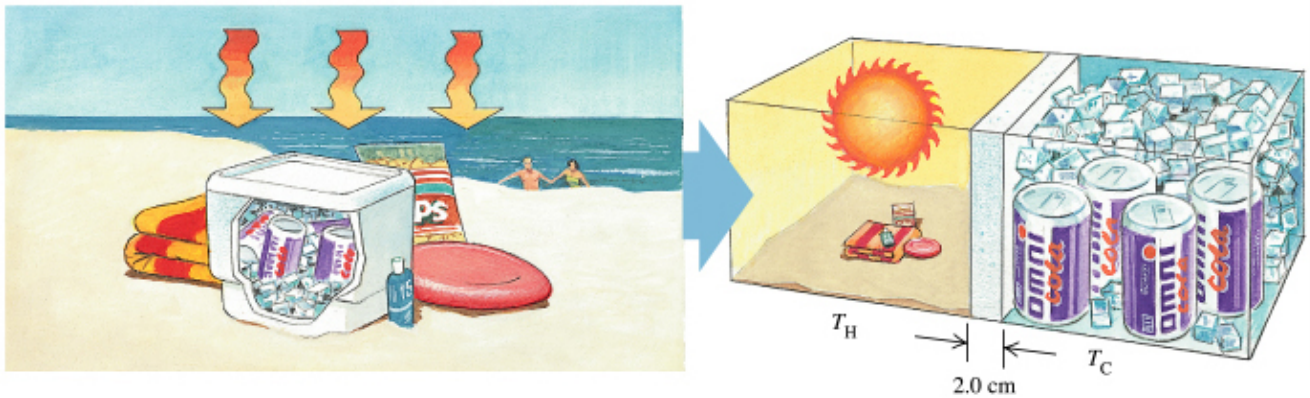
Example. What is the rate at which ice melts inside a 2 cm thick styrofoam cooler. ($k=0.01\text{W/m.K}$ from table 15-5) with an area is 0.8m^2 given the outside temperature is 30°C ?

$$H = kA \frac{T_H - T_C}{L}$$

$$H = (0.01\text{W} / \text{m.K})(0.80\text{m}^2) \frac{(30^\circ\text{C} - 0^\circ\text{C})}{0.02\text{m}} = 12\text{W} = 12\text{J} / \text{s}$$

The rate at which ice melts is given by

$$R = H/L_f = \frac{12\text{J} / \text{s}}{3.34 \times 10^5 \text{J} / \text{kg}} = 3.6 \times 10^{-5} \text{kg} / \text{s}$$



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Thermal resistance

One can rewrite the heat flow equation so that it is similar to Ohm's law for electricity where the voltage drop (ΔV) across a resistor with current I is given by:

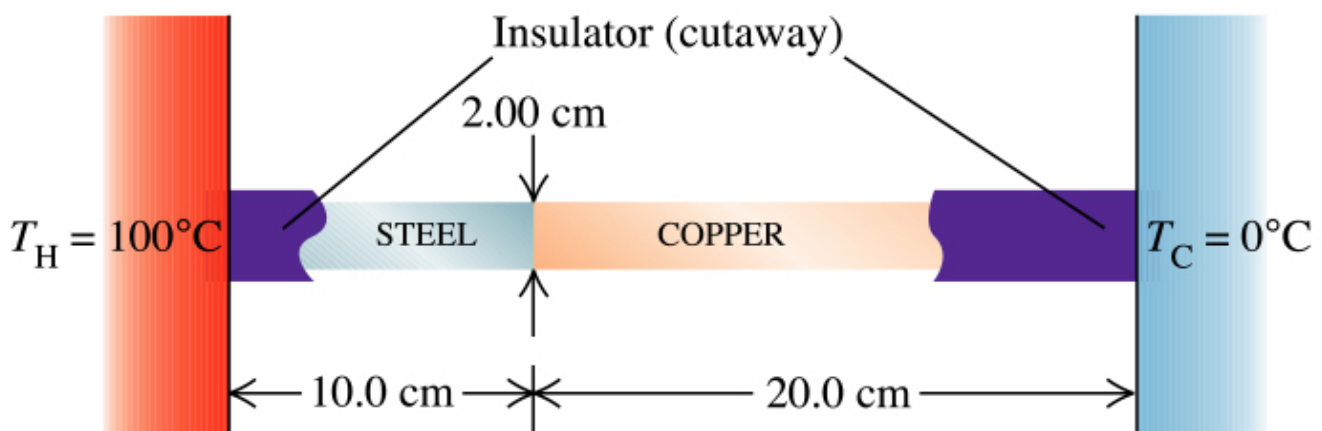
$$\Delta V = IR_e$$

In analogy:

$$\Delta T = HR_t; \text{ where the thermal resistance}$$

$$R_t \equiv L/kA$$

Note this definition of thermal resistance is different than in the text where $R=L/k$. We will call this the R-factor or R_f (see below). Thermal resistances of two substances add in series just the way electrical resistances do. Suppose one has two bars end to end between the hot and cold reservoirs. What is the heat flow?



Let T_x be the temperature at the interface between two materials labeled a and b. The key point is that **in the steady state the heat current flowing in bar a is the same as in bar b** in order for energy to be conserved.

$$H_a = H_b \equiv H \text{ by energy conservation}$$

$$T_H - T_X = H_a R_a; \text{ where } R_a = L_a/k_a A_a$$

$$T_X - T_C = H_b R_b; \text{ where } R_b = L_b/k_b A_b$$

Adding the last two equations with $H=H_a=H_b$ yields:

$$T_H - T_C = H(R_a + R_b)$$

Now you can solve for H and T_x . **Exercise** this for repeat for 3 bodies a,b and c. It can be generalized to n different substances in series.

$$T_H - T_C = HR_{\text{total}} \text{ where}$$

$$R_{\text{total}} = R_1 + R_2 + R_3 + \dots R_n$$

R- factor (R_f)

In Industry the R- factor is defined as:

$$R_f = \frac{L}{k} = AR_t \text{. so that}$$

$$H = A(T_H - T_C)/R_f$$

This gives a way to rate the insulation value of a certain thickness of a material. The SI units of R_f are $m^2 \cdot K/W$. whereas in commercial applications they are in $ft^2 \cdot ^\circ F \cdot h/Btu$ but usually quoted without units. e.g. a 6 inch layer of fiberglass has an R-factor of 19 meaning 19 $ft^2 \cdot ^\circ F \cdot h/Btu$.

Heat current through several layers of different substances is obtained by adding the R-factors just like thermal resistances, R_t .

$$R_f^{\text{tot}} = R_f^1 + R_f^2 + \dots R_f^n$$

$$H = \frac{A(T_H - T_C)}{R_f^{\text{tot}}}$$

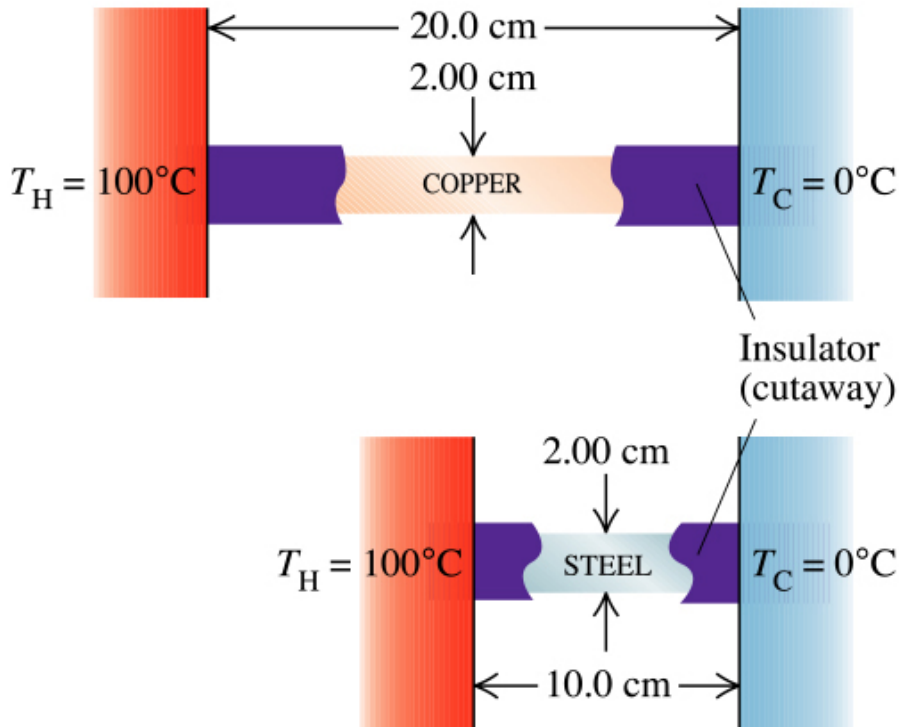
Example: What is the heat flow through a wall panel 2ft wide and 8ft high composed of 6in fibreglass ($R=19$) and 3/8 in gypsum ($R=0.32$) with 20 $^\circ C$ on one side and 0 $^\circ C$ on other?

$$H = \frac{16ft^2 36^\circ F}{(19.0 + .32)ft^2 \cdot ^\circ F} Btu/h = 29.8 Btu/h = 8.7W$$

$$1 \frac{Btu}{h} = \frac{1055J}{3600s} = 0.29W$$

Thermal resistors in parallel

Suppose the two resistors are in parallel as follows



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$$\begin{aligned}
 H &= H_{\text{Cu}} + H_{\text{st}} \\
 &= k_{\text{Cu}} A_{\text{Cu}} \frac{(T_H - T_C)}{L_{\text{Cu}}} + k_{\text{st}} A_{\text{st}} \frac{(T_H - T_C)}{L_{\text{st}}} \\
 &= (T_H - T_C) \left[\frac{1}{R_{\text{Cu}}} + \frac{1}{R_{\text{st}}} \right] = \frac{T_H - T_C}{R_{\text{tot}}} \quad \text{where} \quad \frac{1}{R_{\text{tot}}} = \frac{1}{R_{\text{Cu}}} + \frac{1}{R_{\text{st}}}
 \end{aligned}$$

In other words thermal resistors in parallel add like electrical resistors in parallel. Put in the numbers using table 15-5 where $k_{\text{Cu}}=385 \text{ W/m.K}$ and $k_{\text{st}}=50.2 \text{ W/m.K}$

Convection

Only liquids and gases can transfer heat by convection since motion of atoms/molecules is required. In general there is no simple theory for convection. Some experimental facts are:

1. The heat flow from convection scales with area.
2. The atoms of a gas/liquid near the solid surface are slowed by viscosity. This leads to a quasi-stationary layer of gas near the surface presents a surface resistance to heat flow from convection.

The layer is decreased if the gas/liquid is forced.

Example: Redo the calculation of heat flow from the wall panel taking into account the R-factor for stationary air (e.g. inside a house) is $R_{f,in}=0.68$ whereas for blowing air(outside) $R_{f,out}=0.17$.

$$H = \frac{16\text{ft}^2 36^\circ\text{F}}{(19.0 + 0.32 + 0.68 + 0.17)\text{ft}^2 \cdot ^\circ\text{F}} \text{Btu/h} = 28.5\text{Btu/h} = 8.3\text{W}$$

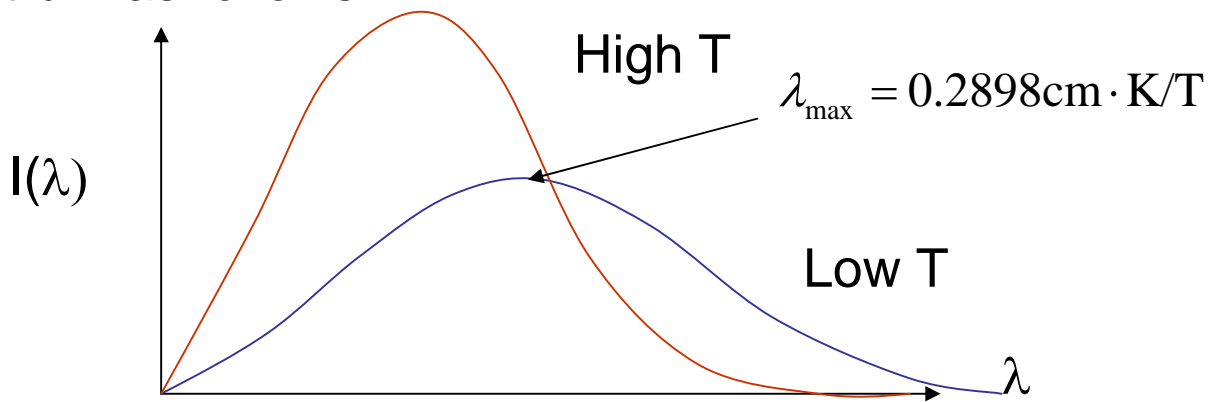
compared to 8.7 W without the surface resistance.

Radiation

Any body at finite T will radiate electromagnetic energy (light) even in a vacuum. This is another means by which energy may be transferred from a hot body to a cold body. Consider a sphere at $T=T_s$.

T_s

The body emits so called black body radiation with a spectrum as follows:



The radiation emitted for a body of surface area A is given by the **Stefan Boltzman law**:

$$H = Ae\sigma T^4$$

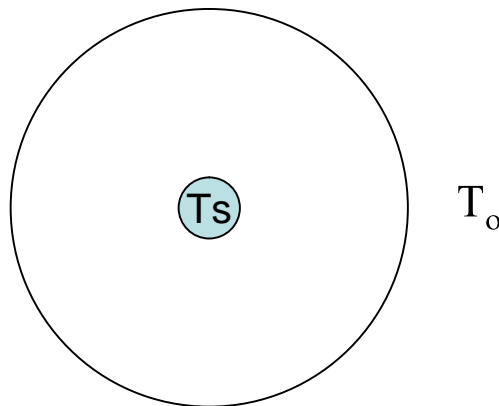
where T is **absolute temperature in K**,
 e is the emissivity, and σ is the Stefan Boltzman constant equal to:

$$\sigma = 5.67051(19) \times 10^{-5} \text{W/m}^2 \cdot \text{K}^4$$

Radiation Cont'd

Emissivity e is a property of the material and equal to a number between 0 and 1. A black body, which by definition absorbs everything, has $e=1$ whereas reflective materials have e values less than 1.

Now suppose the body is surrounded by another body at temperature T_o which radiates onto the inner sphere.



If $T_o = T_s$ radiation emitted by the inner sphere must equal to the radiation absorbed, If the inner and out spheres are not at the same T then the net radiation given off by the inner sphere is:

$$H_{net} = Ae\sigma T_s^4 - Ae\sigma T_o^4 = Ae\sigma(T_s^4 - T_o^4)$$

Example: Consider a tungsten sphere ($e=0.35$) and radius 1.5 cm in a large evacuated enclosure with walls at 26.8°C . What power is needed to keep the temperature of the tungsten sphere at 3000K ?

Example (Cont'd)

$$\begin{aligned} H &= A\sigma e(T_s^4 - T_0^4) \\ &= 4\pi(1.5 \times 10^{-2} \text{ m})^2 \times 5.67 \times 10^{-8} \text{ W} / (\text{m}^2 \cdot \text{K}^4) \times 0.35 [3000^4 - 300^4] \text{ K}^4 \\ &= 4545 \text{ W} \end{aligned}$$

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