Module 2, Lesson 2

Heat Pumps and Refrigerators

Objective: By the end of this lesson you will be able describe the difference between a heat engine and a heat pump/refrigerator and be able to differentiate between the efficiency and the coefficient of performance.

We will also look at how heat pumps can be more effective at heating our houses than an electric heater.

Introduction

As we've seen in the previous lesson, a critical aspect of our ability to survive is the environment we live in. As you know well, we regulate our environment by living in houses that have climates that we can more or less control. If it's too hot we run the air conditioner to cool it down, and if it's too cold we turn on the heater. We can also humidify or dehumidify our air to fit our preferences.

In this lesson we'll discuss some of the more formal aspects of heating and cooling a house, namely, the use of heat pumps and air conditioners, which are really just engines that use work to push air around.

First, we'll discuss the heat engine, which uses thermal energy to generate work. The workings of a heat engine are governed by the first a second laws of thermodynamics, which constrain the engine's energy usage and efficiency. For our discussion it will be helpful to remember the first an second laws of thermodynamics.

The first law of thermodynamics can be stated in terms of the heat energy transferred into the system,

$$Q = W_{\mathrm{s}} + \Delta E_{\mathrm{th}}$$

This simply states that the heat that goes into the system is either stored internal energy ΔE_{th} or is used by the system to do work W_{s}. I should point out that the first law is usually written in terms of the work done by the surroundings $W = -W_s$. The work done by the system is simply the area under a PV curve on a PV diagram. The first law is really just a fancy way of saying "you can't win, you can only break even."

The second law of thermodynamics is the idea that entropy tends to increase. Depending on how comfortable you are with thermodynamics, entropy may seem like a mysterious and nebulous concept that's often stated to be the disorder of a system, but I assure you that it's a real, measurable quantity. For this lesson we'll describe the change in entropy as ratio of the heat added to a system over the temperature of the system

$$\Delta S = \frac{Q}{T}$$

If the temperature is changing we can rewrite the heat in terms of the material's specific heat capacity $Q = C_V \Delta T$. In terms of heat engines the second law tells us that the entropy of the engine plus the entropy of its surrounding can increase, but not decrease. This is a fancy way of dashing your hopes by saying "you can't even break even." I will also point out that the second law implies that heat can only flow from higher temperature to lower temperatures, which will also be useful for discussing heat engines.

1. Heat Engines

You may be familiar with the concept of the heat engine, which uses cyclical processes to turn thermal energy into work. A cyclic process is one in which the system periodically returns to its initial state. The first practical heat engine was the steam engine, which heralded the beginning of the industrial revolution.

Most of the heat engines we'll discuss use some type of gas, such as steam, to do the work, so we'll start by considering the PV (pressure-volume) graph of a thermodynamic process. For an ideal gas PV = nRT, which means that changes in the pressure and volume mean that the temperatures of the system change. Figure 1 shows how to extract work from a cyclic thermodynamic process.

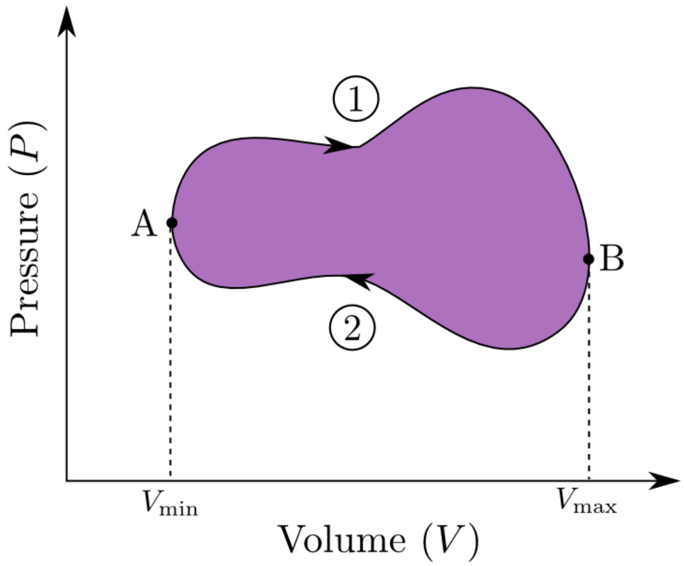


Figure 1. Consider a thermodynamic process drawn on this pressure-volume diagram. The process can be broken into two parts. Path 1 takes the system from point A to point B, from its minimum volume to its maximum volume. Because the system is expanding along Path 1, and is the result of the gas expanding, the area under this curve (purple + white) is the work done by the system W_s. The area under Path 2 (white), which goes from B to A and takes the system from its maximum volume to its minimum volume, is the work done on the system by the surroundings. The difference between the area of these two curves, shaded purple, is then the net work done during one cycle of the heat engine.

An important consequence of the cyclic nature of the heat engine is that the system eventually returns to its initial state, which means that after one cycle there is no net change in internal energy $\Delta E_{th} = 0$. After one cycle the first law of thermodynamics says that the work done by the system is simple the heat that enters the system,

$$Q = W_{\text{s}}$$

Another way to look at heat engines is to consider two heat reservoirs, one hot with a temperature T_H and one cold with a temperature T_C, as the source of thermal energy the engine uses to do work. A key assumption is that these hot and cold reservoirs don't change temperature when energy is added or taken away fro them. This is an idealization, as the temperature of any system will change when its energy changes, but it works quite well if the energy in the reservoirs is much larger than the energies involved in the process. For example, dropping an ice cube in the ocean can hardly change the temperature of the ocean.

We will define two quantities:

$$Q_{\text{H}} = \text{the amount of heat transferred to or from a hot reservoir}$$
$$Q_{\text{C}} = \text{the amount of heat transferred to or from a cold reservoir}$$

The sign of these quantities are given by the first law and how the change the heat into the system. Figure 2 demonstrates how these quantities can be used to get work out of an engine.

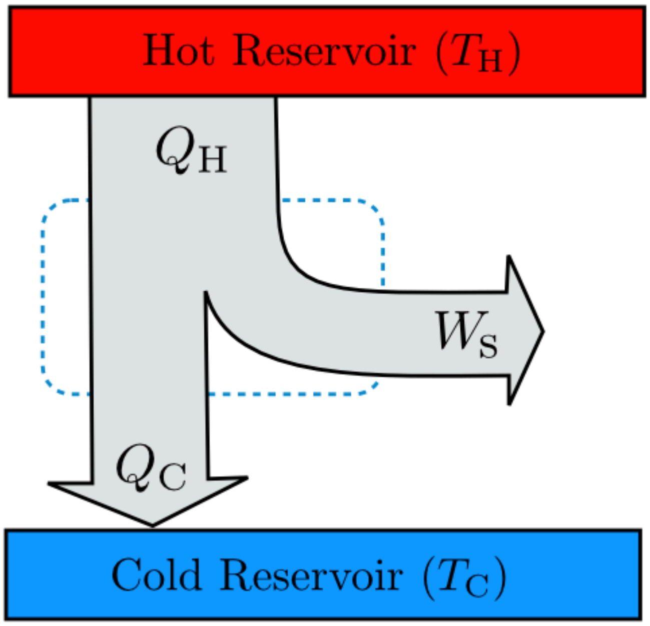
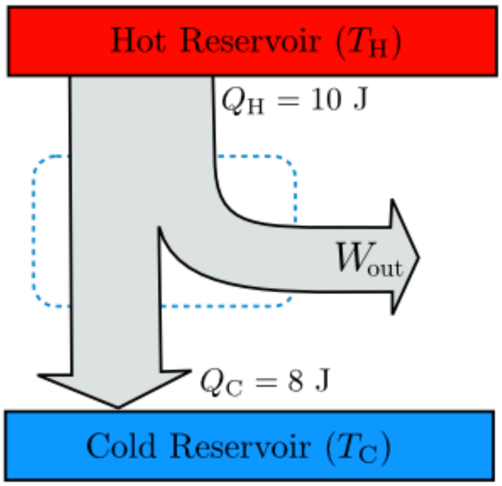


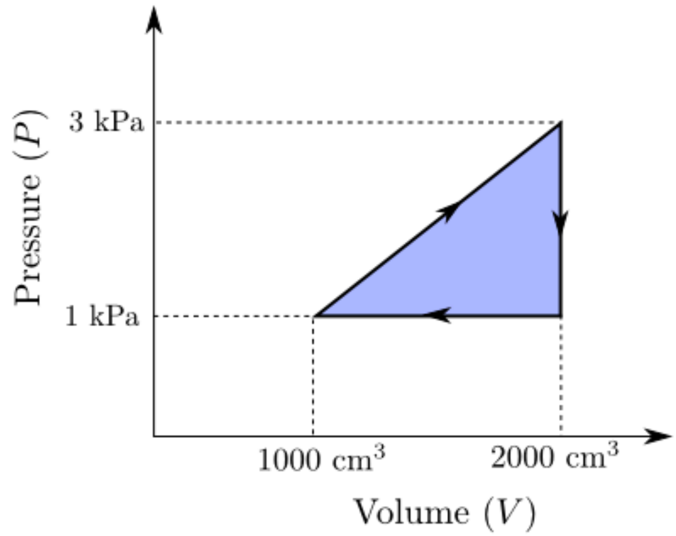
Figure 2. A diagram showing the heat reservoirs, the system as a blue dotted line, and the flow of energy. Heat from the hot reservoir Q_H enters the system and heat Q_C leaves the system and flows into the cold reservoir. The net heat of the system is given by the heat in minus the heat out, $Q = Q_H - Q_C$. Because of the system is cyclic this net heat is equal to the work done by a single cycle of the engine.

We are now ready to determine the net heat produced by a single cycle of the engine. After a single cycle the net heat gained by the system is the heat input from the hot reservoir, usually from burning some kind of fuel, minus the heat lost to the cold reservoir as waste heat. Because the change in internal energy is zero, this heat must equal the net work done by the system in one cycle,

$$Q = W_{\text{out}} = Q_{\text{H}} - Q_{\text{C}}$$

Question: Consider the two engines depicted in the figure below. Calculate the work done by the engine depicted using heat reservoirs minus the work done by the engine depicted using a PV diagram (i.e., your answer will be negative if the engine on the right does more work).





Answer: 2J - 2J = 0 J.

Congratulations! You've made it though the first step of learning about heat engines. At this point all we've used is the first law of thermodynamics, which is really just conservation of energy. We have yet to use the second law, which will come up when we talk about the efficiency of a heat engine.

Efficiency of a heat engine

This has all been a little abstract, but bear with me, we're slowly getting to how this relates to heating and cooling houses. Ideally we would like our engine to do the most amount of work while using using the least amount of energy possible. We can quantify this idea by defining the efficiency as

$$\eta \equiv \frac{\text{benefit}}{\text{cost}} = \frac{W_{\text{out}}}{Q_{\text{H}}}$$

which is the benefit/cost ratio. The first law of thermodynamics let's us rewrite this in terms of just the heat,

$$\eta = \frac{Q_{\text{H}} - Q_{\text{C}}}{Q_{\text{H}}} = 1 - \frac{Q_{\text{C}}}{Q_{\text{H}}}$$

Now, the second law of thermodynamics says that the entropy of the system plus its surroundings must increase. If we think of the cycle of an engine, the heat added to the system from the hot reservoir increases the system's entropy by S_{in}. The heat lost from the system to the cold reservoir increases the entropy of the surroundings by S_{out}. Because the system has returned to its original state, the second law tells us that the entropy out must be at least as much as the entropy in. Using our definition of entropy from the introduction we can write this statement in terms of the heat leaving and entering the reservoirs and the temperatures of the reservoirs,

$$S_{\text{out}} \geq S_{\text{in}} \Rightarrow \frac{Q_{\text{C}}}{T_{\text{C}}} \geq \frac{Q_{\text{H}}}{T_{\text{H}}}$$

With this inequality we can rewrite our efficiency in terms of the temperatures of the reservoir

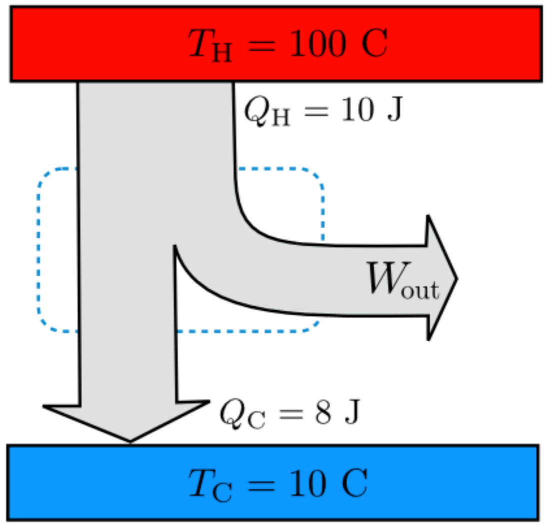
$$\eta \leq 1 - \frac{T_{\text{C}}}{T_{\text{H}}}$$

It might not seem like much, but this is one of the most important of consequences of thermodynamics. What it tells us its that the efficiency of any heat engine is limited by the temperature of the heat reservoirs. Step back and ponder how profound this is. With a simple argument we have limited the efficiency of every possible cyclical engine configuration.

Now that we've found the best efficiency, you might wonder what is the lucky heat engine that has this efficiency? Well, prepare to be disappointed. The heat engine that has this efficiency is called the Carnot Cycle, and it only exists in theory. If we look above at how we derive the bound of the efficiency, the limiting step is that the entropy out must be greater or equal to the entropy in. The efficiency is greatest when the entropies are equal, which means that the change in entropy of the system plus the surroundings is zero.

This means that the process is reversible. In order to make a reversible engine the Carnot Cycle has a step that takes an infinite amount of time: not a very useful engine. Even engines that approximate a Carnot cycle must work so slowly that they output nearly no power. Real life engines don't come near to this efficiency. When we talk about generating energy we'll talk about this more.

Question: Consider the engine below. How does the efficiency of this engine compare to the theoretical maximum efficiency given these heat reservoirs? State you answer in terms of the Carnot efficiency minus the efficiency of the depicted engine.



Answer: The efficiency of the depicted engine is 2J/10J = 0.2. The Carnot efficiency is 1-283/373 = 0.241. The difference is then 0.241 - 0.2 = 0.041.

2. Heat Pumps and Refrigerators

We've talked a lot about how heat engines can be used to generate work, but how about using work to move heat around? It's easy to move heat from a hot place to a cold place, systems want to do this naturally. But what about moving heat from cold places to hot places? In your house you likely have a refrigerator, and depending on the climate of where you live you may even even an air conditioner.

The goal of these is to remove heat from a region to keep it cold. You may also have a heat pump, which uses work to add heat to a region. Both of these are really the same process and, as you can see in Figure 3, are just the reverse of a heat engine. In fact, if we take the process in Figure 1 and reverse the path (flip the arrows) we get a refrigerator. The net work done by the system is negative, which means that the surroundings must supply the work.

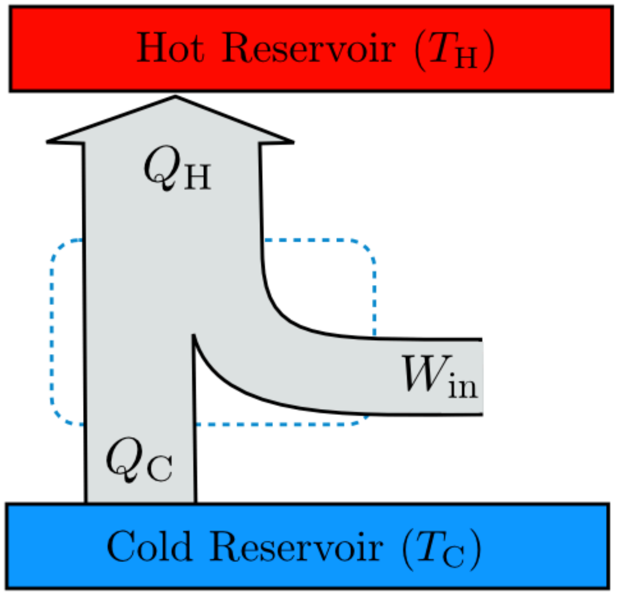


Figure 3. This process uses work to move heat from the cold reservoir to the hot reservoir. It describes both a heat pump and a refrigerator. Which it describes depends on the quantity you're interested in. If you want to keep something warm you're interested in Q_H and you have a heat pump, if you want to keep something cold you're interested in Q_C and you have a refrigerator.

Just as we defined efficiency for heat engines, we would like to somehow quantify how well the refrigerator is doing its job. The relevant number is still the benefit/cost ratio, but what we put in and what we get out are different. We call this ratio the coefficient of performance,

$$\mathrm{COP} = \frac{\text{benefit}}{\text{cost}} = \frac{Q_C}{W_{in}}$$

If you replace Q_C with Q_H you get the the coefficient of performance of a heat pump. What the COP tells us is how much heat is moved for a given amount of work.

Unlike the efficiency, which must be smaller than one, the COP can be larger than one, but it's still limited by the second law of thermodynamics. Following an argument similar to the heat engine, after one cycle, the entropy in is Q_C/T and the entropy out is Q_H/T. Because the system returns to its original state, the entropy of the surroundings must be greater or equal to zero, which means that

$$S_H \geq S_C \Rightarrow \frac{Q_H}{T_H} \geq \frac{Q_C}{T_C}$$

which makes sense, because this process is the opposite of a heat engine. We can then bound the COP for a refrigerator as

$$\text{COP} \leq \frac{T_\text{C}}{T_\text{H} - T_\text{C}}$$

The theoretical maximum COP for a heat pump can be found by replacing T_C in the above formula by T_H.

Though the COP is an ideal dimensionless quantity to measure the effectiveness of a heat pump or refrigerator, it's rarely used in industry. Much more common is the Energy Efficiency Ratio (EEF), which is given by

$$\text{EEF} = \frac{\text{power moved (BTU/hour)}}{\text{electrical input (Watts)}}$$

An EEF can be converted into a COP simply by using the conversion 1 BTU/hour = 0.292 Watts. Therefore,

$$\text{COP} = 0.292 \text{ EEF}$$

We are now ready to talk about some concrete applications of heat pumps.

3. How to make better use of energy

Great! So what does this all mean for heating your house? Let's think about how we use a heat pump to get get more heat from our electricity usage. Suppose we start with 1 J of electrical energy. We'll use this energy to heat our house, which is a toasty 20 C, while the temperature outside is a chilly 5 C.

We can choose to use electrical energy to run a radiative heater, which is essentially a giant resistor. All of the energy that goes into the resistor is dissipates as heat, so our 1 J of electric energy gets us 1 J of heat. Not to bad. Because of this it's often stated that an electrical heater has an efficiency of 1, which isn't quite right because no work is being done. The correct quantity is actually the COP,

$$\text{COP} = \frac{Q_\text{T}}{W_\text{in}} = \frac{1 \text{ J}}{1 \text{ J}} = 1$$

We get 1 J of heating for every Joule of electrical energy we use. The radiative heater doesn't use heat reservoirs so the temperature inside and outside our house doesn't enter our considerations.

Can we make better use of this 1 J of electrical energy? If we use a heat pump we can use some of the thermal energy outside our house to heat inside our house.

Question: Using the temperatures from above, if the inside of our house is the hot reservoir the heat pump uses the outside is the cold reservoir, what is the theoretical best COP achievable?

Answer: COP = 293/(293-278) = 19.5! This is much higher than the radiative heater, but is it realistic?

The theoretical COP for the heat pump is very promising. With it we could get 20 J of heat for our 1 J of electrical energy. For a more realistic we can check the Energy Star ratings for heat pumps from the Canadian Office of Energy Efficiency where they use EER to measure the effectiveness. We see that lowest Energy Star ratings is around around EEF = 11, which corresponds to a COP = 3.2. This is much lower than our theoretical value, but it's still about three times better than the value obtained for a radiative heater.

Summary

The tools we've developed here are powerful. They will not only heavily influence our discussion in this module, but for the rest of the course, particularly when we look at the feasibility of difference forms of alternative energy production.

Resources

For thsoe interested in the more formal aspects of thermodynamics I suggest An Introduction to Thermal Physics by Schroeder [http://physics.weber.edu/thermal/].