Topological Currents in Neutron Stars

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What is a topological vector current?

They are non-dissipative currents that appear due to topological effects.

Can be derived using
- anomalous effective lagrangians\(^1\)
- index theorems\(^2\)

Topological vector currents have the form

\[
\langle j \rangle = (\mu_l - \mu_r) \frac{e\Phi}{2\pi^2}.
\]

Φ = magnetic flux

\(\mu_l/\mu_r = \) left and right handed electron chemical potential.

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Why do we expect currents in Neutron Stars?

Basic neutron star properties are,

- radius $R \sim 10$ km.
- cold $T_{\text{star}} \sim 10^9$ K $\ll T_{\text{Fermi}} \sim 10^{12}$ K.
- large magnetic field $B \sim 10^{12}$ G. *
- dense $10^{14}$ g/cm$^3 \rightarrow \mu_e \sim 80$ MeV. *
- equilibrium processes break P symmetry, e.g., *

  $$n + n \rightarrow n + p + e^- + \bar{\nu}_e$$
  $$n + p + e^- \rightarrow n + n + \nu_e$$

* All the requirements for the current exist.
How does the helicity manifest itself?

For an infinite system detailed balance washes out the helicity.

- same number of left hand electrons are created as are destroyed.
- $\mu_l = \mu_r$, there is no current.

The neutron star is a finite system and electrons are not in equilibrium

- the mean free path of electron with respect to weak interaction $\gg R$.
- non-dissipating current allows electrons to leave the star before they decay.
- overall helicity appears.
Estimating the magnitude of the current

There are two problems

- the equation shown earlier for the current is too idealised
- flux penetrates star non uniformly

Assume the magnetic field is uniform, the flux is the same.

- electrons created with parity $P$ leave the system due to the current
- calculate the creation rate of electrons for entire star $w/\Omega \cdot V_{\text{star}}$
- normalise to the number of vortices $N_v$

We arrive at,

$$\langle j \rangle = P_{\text{asym}}(B, \mu, T) \frac{w(B, \mu, T)}{\Omega} \frac{V_{\text{star}}}{N_v}$$
What processes create and destroy electrons?

Many processes may be dominant inside a neutron star

- direct Urca \[ n \rightarrow p + e^- + \bar{\nu}_e \]
- modified Urca \[ n + n \rightarrow n + p + e^- + \bar{\nu}_e \]
- kaon condensate \[ \langle K^- \rangle + n \rightarrow n + e^- + \bar{\nu}_e \]
- quark Urca \[ d \rightarrow u + e^- + \bar{\nu}_e \]

The electrons created are mostly left handed, \( P = 0.85 \).

The mean free path of the electron for these processes is larger than the neutron star.
How big is the current?

The currents are very sensitive to temperature.

Kaon condensate, Quarks, and Direct Urca

\[ \langle j \rangle \sim 10^{-9} \left( \frac{T}{10^9 \text{ K}} \right)^5 \text{ MeV} \]

Modified Urca

\[ \langle j \rangle \sim 10^{-14} \left( \frac{T}{10^9 \text{ K}} \right)^7 \text{ MeV} \]

per quantum unit of flux
(or per single type-II vortex).
Neutron stars have been “kicked”;
- progenitor has proper motion $\sim 30$ km/s.
- neutron star motion $\sim 200$ km/s, with some $> 1000$ km/s.

Electrons exit star and transfer momentum
- $1000 \left( \frac{T}{10^9 \text{K}} \right)^5$ years to reach 1000 km/s
- neutron stars have thick crusts, quarks
  stars have thin crusts
  $\Rightarrow$ may be a way to tell them apart
- there is now an extra cooling mechanism
Applications: Precession

Neutron stars precess

- the angle of precession conflicts with type-II vortices.
- a current \( j > \frac{1}{4e\lambda} \) running along type-II vortices makes them act like type-I\(^3\)
- at \( T \sim 5 \cdot 10^9 \) K the current can satisfy this inequality
- may be too hot for superconductivity, thus no vortices.

\(^3\)Charbonneau and Zhitnitsky, astro-ph/0701308 (2007)
Neutron stars have large poloidal magnetic fields $B \sim 10^{12}$ G.

- for stability a toroidal field of the same order is needed
- at $T \sim 10^8$ the modified Urca current induces $B \sim 10^{13}$ G (this includes counter field for Meissner effect)
- other currents produce giant fields $10^{19}$ G (almost completely destroys superconductivity)
If there is one fact to take away from this and tell your friends it is,

A current travels along the magnetic field in neutron stars.