Topological Currents in Neutron Stars

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February 2009
1 Introduction
   - What is a topological current?
   - Why do we expect them in neutron stars?

2 Estimating the magnitude of the current.

3 Applications in neutron stars
What is a topological vector current?

They are non-dissipative currents that appear due to topological effects.

Can be derived using

- anomalous effective lagrangians\(^1\)
- index theorems\(^2\)

Topological vector currents have the form

\[
\langle j \rangle = (\mu_l - \mu_r) \frac{e\Phi}{2\pi^2}.
\]

\(\Phi = \) magnetic flux

\(\mu_{l/r} = \) left and right handed electron chemical potential.

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Why do we expect currents in Neutron Stars?

Basic neutron star properties are,

- radius $R \sim 10$ km.
- cold $T_{\text{star}} \sim 10^9$ K $\ll T_{\text{Fermi}} \sim 10^{12}$ K.
- large magnetic field $B \sim 10^{12}$ G. *
- dense $10^{14}$ g/cm$^3$ $\rightarrow \mu_e \sim 80$ MeV. *
- equilibrium processes break P symmetry, e.g., *

\[
\begin{align*}
n + n & \rightarrow n + p + e^- + \bar{\nu}_e \\
n + p + e^- & \rightarrow n + n + \nu_e
\end{align*}
\]

* All the requirements for the current exist.
Estimating the magnitude of the current

1. Finite system, electrons not in equilibrium
   - the mean free path of electron with respect to weak interaction $\gg R$.
   - electron leaves the star before it decays.

2. Equation from earlier is too idealised and magnetic flux structure is non-trivial.

3. Assume the magnetic field is uniform,
   - calculate the creation rate of electrons with parity $P$ for entire star $w/\Omega \cdot V_{\text{star}}$
   - normalise to the number of vortices $N_v$

We arrive at,

$$\langle j \rangle = P_{\text{sym}}(B, \mu, T) \frac{w(B, \mu, T) V_{\text{star}}}{\Omega} \frac{\Omega}{N_v}$$
How big is the current?

The dominant electron producing process is widely debated. Calculate them all.

Kaon condensate, Quarks, and Direct Urca

\[ \langle j \rangle \sim 10^{-9} \left( \frac{T}{10^9 \text{ K}} \right)^5 \text{ MeV} \]

Modified Urca

\[ \langle j \rangle \sim 10^{-14} \left( \frac{T}{10^9 \text{ K}} \right)^7 \text{ MeV} \]

per quantum unit of flux
(or per single type-II vortex).
Neutron stars have been “kicked”;
- progenitor has proper motion $\sim 30$ km/s.
- neutron star motion $\sim 200$ km/s, with some $> 1000$ km/s.

Electrons exit star and transfer momentum
- $1000 \left( \frac{T}{10^9 \text{ K}} \right)^5$ years to reach 1000 km/s
- neutron stars have thick crusts, quarks
  stars have thin crusts
  $\Rightarrow$ may be a way to tell them apart
- there is now an extra cooling mechanism
Neutron stars precess

- the angle of precession conflicts with type-II vortices.
- a current $j > \frac{1}{4e\lambda}$ running along type-II vortices makes them act like type-I$^3$
- at $T \sim 5 \cdot 10^9$ K the current can satisfy this inequality
- may be too hot for superconductivity, thus no vortices.

Neutron stars have large poloidal magnetic fields $B \sim 10^{12}$ G.

- for stability a toroidal field of the same order is needed
- at $T \sim 10^8$ the modified Urca current induces $B \sim 10^{13}$ G (this includes correction for Meissner effect)
- other currents produce giant fields $10^{19}$ G (almost completely destroys superconductivity)
If there is one fact to take away from this and tell your friends it is,

A current travels along the magnetic field in neutron stars.