Phase diagram of dense QCD with and without neutrino trapping

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Outline

- Introduction and motivation

- Results for the phase diagram(s) of dense QCD
  - Self-consistent treatment of quark masses
  - Effect of neutrino trapping
  - Problems, limitations, etc.

- Current “state of the art”

- Conclusions

- Outlook

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Dense baryonic matter in Nature

Compact (neutron) stars

- Radius:
  \( R \approx 10 \text{ km} \)
- Mass:
  \( 1.25 M_\odot \lesssim M \lesssim 2M_\odot \)
- Core temperature:
  \( 10 \text{ keV} \lesssim T \lesssim 10 \text{ MeV} \)
- Surface magnetic field:
  \( 10^8 \text{ G} \lesssim B \lesssim 10^{14} \text{ G} \)

What is the state of matter at the highest stellar densities, \( \rho_c \gtrsim 5\rho_0 \)?
The region of prime interest is
\[ \mu \gtrsim \Lambda_{QCD} \gtrsim T \]

- So far, there are no reliable lattice results at \( \mu \gtrsim \Lambda_{QCD} \)
- Effective models have a limited predictive power
- Effects of charge neutrality and \( \beta \) equilibrium are not under control
- Difficulties in determining stable ground states

So, how hard could it be?
Ground state of dense quark matter

Noninteracting quarks:

(i) Deconfined quarks ($\mu \gg \Lambda_{QCD}$)
(ii) Pauli principle ($s = \frac{1}{2}$)

⇒ Cooper instability

Color superconductivity

\[ \langle (\bar{\Psi}^C)_{\alpha}^i \gamma_5 \Psi_{\beta}^j \rangle \neq 0 \]

Interacting quarks:

(i) Effective models ($\mu \gtrsim \Lambda_{QCD}$)
(ii) One-gluon exchange ($\mu \gg \Lambda_{QCD}$)
Unconventional Cooper pairing

- Wave function of a spin-0 Cooper pair:
  \[(|\bullet\bullet\rangle - |\bullet\bullet\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes (|u_p, d_{-p}\rangle - |d_p, u_{-p}\rangle)_{1,\bar{3}}\]
- In \(\beta\)-equilibrium, quarks have non-equal Fermi momenta:
  \(p_F^{(u)} \neq p_F^{(d)} \neq p_F^{(s)}\)

Charge neutrality \((N_f = 2)\)

\[
\begin{align*}
\mu_d &= \mu_u + \mu_e \\
2n_u - \frac{1}{3}n_d - n_e &= 0
\end{align*}
\]

\[
\delta \mu \equiv \frac{p_F^{(d)} - p_F^{(u)}}{2} = \frac{\mu_e}{2}
\]

Color neutrality \((N_f = 3)\)

\[
\begin{align*}
m_s &\gg m_u, m_d \\
\text{CFL: strange} &\Leftrightarrow \text{blue}
\end{align*}
\]

\[
\delta \mu \equiv \frac{p_F^{(bd)} - p_F^{(gs)}}{2} \approx \frac{m_s^2}{2\mu}
\]

[Alford,Kouvaris&Rajagopal,hep-ph/0311286]

How does the mismatch \(\delta \mu \neq 0\) affect Cooper pairing?

$$\mathcal{L} = \bar{\psi} (i\slashed{\partial} - \hat{m}) \psi + G_S \sum_{a=0} \left[ (\bar{\psi} \lambda_a \psi)^2 + (\bar{\psi} i\gamma_5 \lambda_a \psi)^2 \right]$$

$$+ G_D \sum_{\gamma,c} \left[ \bar{\psi}^a \gamma_5 \epsilon_{\alpha \beta} \epsilon_{abc} (\psi_C)^b_{\gamma} \right] \left[ (\bar{\psi}_C)^a \gamma_5 \epsilon_{\rho \sigma \gamma} \epsilon_{rs \gamma} \psi_{s}^{\sigma} \right]$$

$$- K \left\{ \text{det} \left[ \bar{\psi} (1 + \gamma_5) \psi \right] + \text{det} \left[ \bar{\psi} (1 - \gamma_5) \psi \right] \right\}$$

$$m_{u,d} = 5.5 \text{ MeV} \hspace{1cm} m_\pi = 135.0 \text{ MeV}$$

$$m_s = 140.7 \text{ MeV} \hspace{1cm} m_K = 497.7 \text{ MeV}$$

$$G_S \Lambda^2 = 1.835 \hspace{2cm} \Rightarrow \hspace{2cm} m_{\eta'} = 957.8 \text{ MeV}$$

$$K \Lambda^5 = 12.36 \hspace{2cm} f_\pi = 92.4 \text{ MeV}$$

$$\Lambda = 602.3 \text{ MeV} \hspace{2cm} m_\eta = 514.8 \text{ MeV}$$
General approach

Quark chemical potentials:

$$\mu_{ab}^{\alpha\beta} = \left( \mu \delta^{\alpha\beta} + \mu_Q Q_f^{\alpha\beta} \right) \delta_{ab} + \left[ \mu_3 (T_3)_{ab} + \mu_8 (T_8)_{ab} \right] \delta^{\alpha\beta}$$

Dynamically generated quark masses:

$$\hat{M} = \text{diag}_f (M_u, M_d, M_s), \quad \text{with} \quad M_\alpha = m_\alpha - 4G_S \sigma_\alpha + 2K \sigma_\beta \sigma_\gamma$$

Allowed condensates:

$$\Delta_c \sim \epsilon^{\alpha\beta\gamma} \epsilon_{abc} \langle (\bar{\psi}_C)^a_\alpha i\gamma_5 \psi^b_\beta \rangle \quad \text{(no sum over color “c”)}$$

$$\sigma_\alpha \sim \langle \bar{\psi}_\alpha^a \psi_\alpha^a \rangle \quad \text{(no sum over flavor “\alpha”)}$$
Gap equations and neutrality constraints

Pressure:

\[ p = p_L - \frac{1}{4G_D} \sum_{c=1}^{3} |\Delta_c|^2 - 2G_S \sum_{\alpha=1}^{3} \sigma_\alpha^2 + 4K \sigma_u \sigma_d \sigma_s + \frac{1}{2} \ln \det \frac{S^{-1}}{T} \]

Coupled set of 9 equations:

\[ \frac{\partial p}{\partial \sigma_\alpha} = 0 \]

\[ \frac{\partial p}{\partial \Delta_c} = 0 \]

\[ n_Q \equiv \frac{\partial p}{\partial \mu_Q} = 0 \]

\[ n_3 \equiv \frac{\partial p}{\partial \mu_3} = 0 \]

\[ n_8 \equiv \frac{\partial p}{\partial \mu_8} = 0 \]

Note: charge neutrality is enforced locally (no mixed phases allowed)

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Phase diagram, $\mu_{\nu_e} = 0$

(without neutrino trapping)

[Rüster, Werth, Buballa, Shovkovy & Rischke, hep-ph/0503184]

\[ G_D = \frac{3}{4} G_S \] (intermediate coupling) \hspace{1cm} \[ G_D = G_S \] (strong coupling)

**Note:** Gapless phases play little role at strong coupling, $G_D = G_S$

**See also** [Blaschke, Fredriksson, Grigorian, Sandin & Öztaš, hep-ph/0503194]

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Phase diagram, $\mu_{\nu_e} \neq 0$

(with neutrino trapping)

[Rüster, Werth, Buballa, Shovkovy & Rischke, work in progress]

$\mu_{\nu_e} = 200$ MeV

$\mu_{\nu_e} = 400$ MeV

Note: gapless phases play little role already at $G_D = \frac{3}{4}G_S$
Neutrino trapping

$\mu_{L_e} > 0$

inside hot matter

$T \lesssim 40$ MeV

[Rüster, Werth, Buballa, Shovkovy & Rischke, in preparation]
The problem of instabilities

Chromomagnetic instability in the g2SC phase

\[ A = 1, 2, 3 \quad \text{— red solid line} \]
\[ A = 4, 5, 6, 7 \quad \text{— green long-dash line} \]
\[ A = \tilde{8} \quad \text{— blue short-dash line} \]

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Chromomagnetic instability in gCFL phase

Similar results for Meissner screening masses
[Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401]:

\[
\frac{m^2_M(M_s)}{m^2_M(0)} \quad \text{versus} \quad \frac{M^2_s}{\mu_b}
\]

\[
\frac{m^2_M(M_s)}{m^2_M(0)} \quad \text{versus} \quad \frac{M^2_s}{\mu_b}
\]

\(A = 1, 2\) — solid line
\(A = 3\) — short-dashed line
\(A = 8\) — long-dashed line
\(A = 4, 5\) — dashed line
\(A = 6, 7\) — solid line

See also [Wu & Yip, cond-mat/0303185], [Alford & Wang, hep-ph/0501078]
State of the art

(i) $g_{2SC} \rightarrow$ mixed phase  
\[\text{[Reddy & Rupak, nucl-th/0405054]}\]

(ii) $g_{2SC}/g_{CFL} \rightarrow$ crystalline (LOFF) phase  

(iii) $g_{2SC}/g_{CFL} \rightarrow g_{2SC}/g_{CFL} \oplus$ secondary pairing  
\[\text{[D.K. Hong, hep-ph/0506097]}\]

(iv) $2SC \ (\delta\mu < \delta < \sqrt{2}\delta\mu) \rightarrow$ “gluonic” phase with broken rotational SO(3) and $U(1)_{em}$  
Conclusions

- Neutrality and $\beta$-equilibrium strongly affect the properties of dense quark matter
- Phase diagram of neutral dense matter has a very rich structure
- Some features of the QCD phase diagram at $\mu \gtrsim \Lambda_{QCD}$ start to develop
- There is a fundamental problem in current understanding of gapless phases and their instabilities
- Several promising alternatives to gapless phases do exist (e.g., LOFF and mixed phases)
Outlook

- One needs to clarify the precise nature of instabilities

- The “price” of imposing neutrality in the LOFF phase should be studied in detail [Giannakis, hep-ph/0507306]

- The possibility of mixed phases should be subjected to close scrutiny (e.g., along the lines of [Maruyama, et al. nucl-th/0503027])

- The possibility of spontaneously induced currents in gapless phases should be studied (e.g., along the lines of [Huang, hep-ph/0504235])

- One should look into other possible ways of stabilizing phases with unconventional Cooper pairing (e.g., gauge field condensates, or meson condensates, etc.) [Gorbar, et al. hep-ph/0507303]
Collaborator(s)

- Stefan Rüster
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- Dirk Rischke
- Michael Buballa

References

