Neutron Rich Nuclei in Heaven and Earth

Jorge Piekarewicz
Florida State University
Tallahassee, Florida, USA
with B. Todd-Rutel and C.J. Horowitz

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Equation of State I: Generalities

The Bethe-Weizsäcker (BW) Mass Formula:

$$E(Z, N) = a_{\text{vol}} A + a_{\text{surf}} A^{2/3} + a_{\text{coul}} Z^2 / A^{1/3} + a_{\text{symm}} (N - Z)^2 / A + \ldots$$

- Parameters extracted from a fit to thousands of known nuclear masses
- Hidden behind its success is the saturation of the nuclear force
- BW constrains the above parameters at (or near) saturation density:
  $$(a_{\text{vol}}, a_{\text{surf}}, a_{\text{coul}}, a_{\text{symm}}) \simeq (-16, +18, +1, +26) \text{ MeV}$$

BW offers little on the density dependence of the parameters!
Equation of State II: Infinite Nuclear Matter

Recipe to make infinite nuclear matter:

- Turn off the long-range Coulomb force
- Let $Z$, $N$ and $V$ go to infinity with ratios remaining finite:
  \[ \rho = A/V, \quad Y = Z/A, \quad b = \delta = (N-Z)/A, \ldots \]
- Only surviving terms in the thermodynamic limit:
  \[ E(Z, N)/A = a_{\text{vol}} + a_{\text{symm}} b^2 = \epsilon_0 + J b^2 \]

Symmetric vs Asymmetric Matter:

- Expand the total energy per nucleon around $b=0$:
  \[ E(\rho; b)/A = \frac{E(\rho; b=0)}{A} + b \left( \frac{\partial E}{\partial b} \right)_{b=0} + b^2 \left( \frac{\partial^2 E}{\partial b^2} \right)_{b=0} + \ldots \]

Pure neutron matter $\approx$ Symmetric Matter + Symmetry Energy!

Goal: Study the density dependence of the equation of state
Density Functional, Kohn-Sham, MF Theory

Improving the Bethe-Weizsäcker (BW) Mass Formula:

\[ \mathcal{L}_{\text{int}} = g_s \bar{\psi} \gamma \phi - g_v \bar{\psi} \gamma^\mu \psi V^\mu - \frac{g_\rho}{2} \bar{\psi} \gamma^\mu \mathbf{\tau} \cdot \mathbf{b}^\mu \psi - e \bar{\psi} \gamma^\mu \tau_p \psi A^\mu \\
- \frac{\kappa}{3!} (g_s \phi)^3 - \frac{\lambda}{4!} (g_s \phi)^4 + \Lambda_v (g_v^2 V^\mu V^\mu) (g_\rho^2 b^\mu b^\mu) + \frac{\zeta}{4!} g_v^4 (V^\mu V^\mu)^2 \]

- Parameters fitted to a large body of ground-state properties (mostly binding energies and charge radii of many nuclei)
- Ground-state observables computed at the mean-field level
- Formalism is NOT Hartree (Hartree-Fock) theory
- Parameters of the model encode correlations that go beyond two-body (short, long, and pairing correlations in an average way)

Resulting model unlikely to describe correctly NN physics!

Correlating Model Parameters to the Physics:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Constrained by</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g_s, g_v)</td>
<td>Ground state properties of finite nuclei</td>
</tr>
<tr>
<td>(g_\rho)</td>
<td>Ground state properties of heavy nuclei</td>
</tr>
<tr>
<td>(\kappa, \lambda)</td>
<td>Isoscalar giant monopole resonance</td>
</tr>
<tr>
<td>(\Lambda_v)</td>
<td>Neutron radius of heavy nuclei</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>Neutron star structure</td>
</tr>
</tbody>
</table>

- Existent observables insufficient to constrain all parameters
- Determination of neutron radii of neutron-rich nuclei presses!
- Simultaneous mass-radius measurement of neutron stars presses!

Crucial measurements in Heaven and Earth on the horizon!
Accurately Calibrated Parametrizations

The Program:
- Input binding energy and charge radii of doubly magic nuclei
  Solve in self-consistent mean-field approximation
- Compute the linear response of the mean-field ground state
  Solve in self-consistent MF+RPA approximation
- Without any further adjustment, compare to EoS from nuclear collisions
  Up to five times nuclear-matter saturation density
- Without any further adjustment, predict neutron-star structure
  Only physics that neutron stars are sensitive to — is the EoS
  of neutron-rich matter ...

<table>
<thead>
<tr>
<th>A</th>
<th>Observable</th>
<th>Experiment</th>
<th>NL3</th>
<th>NL3_030</th>
<th>FSUGold_000</th>
<th>FSUGold*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Stiff-Stiff</td>
<td>Stiff-Soft</td>
<td>Soft-Stiff</td>
<td>Soft-Soft</td>
</tr>
<tr>
<td>90Zr</td>
<td>( B/A ) (MeV)</td>
<td>8.71</td>
<td>8.69</td>
<td>8.70</td>
<td>8.68</td>
<td>8.68</td>
</tr>
<tr>
<td></td>
<td>( R_{ch} ) (fm)</td>
<td>4.26</td>
<td>4.26</td>
<td>4.27</td>
<td>4.25</td>
<td>4.25</td>
</tr>
<tr>
<td></td>
<td>( R_n - R_p ) (fm)</td>
<td>—</td>
<td>0.11</td>
<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>GMR (MeV)</td>
<td>17.89 ± 0.20</td>
<td>18.62</td>
<td>18.75</td>
<td>17.89</td>
<td>17.98</td>
</tr>
<tr>
<td>208Pb</td>
<td>( B/A ) (MeV)</td>
<td>7.87</td>
<td>7.88</td>
<td>7.89</td>
<td>7.87</td>
<td>7.89</td>
</tr>
<tr>
<td></td>
<td>( R_{ch} ) (fm)</td>
<td>5.50</td>
<td>5.51</td>
<td>5.52</td>
<td>5.51</td>
<td>5.52</td>
</tr>
<tr>
<td></td>
<td>( R_n - R_p ) (fm)</td>
<td>—</td>
<td>0.28</td>
<td>0.20</td>
<td>0.29</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>GMR (MeV)</td>
<td>14.17 ± 0.28</td>
<td>14.32</td>
<td>14.74</td>
<td>13.73</td>
<td>14.04</td>
</tr>
<tr>
<td></td>
<td>GDR (MeV)</td>
<td>13.30 ± 0.10</td>
<td>12.70</td>
<td>13.07</td>
<td>12.79</td>
<td>13.07</td>
</tr>
</tbody>
</table>

* Disclaimer: Gold is referred to the color — not the metal!
Experimental Extraction of $R_n - R_p$

JLAB Experiment 00-003 (03-011) [Michaels, Souder, Urciuoli]:
- Parity Violating Asymmetry in elastic $e^-\text{Pb}$ scattering
- Electroweak (as opposed to hadronic) probe of neutron density
- Weak-vector boson $Z^0$ couples strongly to neutrons
- A clean and accurate measurement of the neutron radius
  1% or 0.05 fm measurement of the neutron radius of $^{208}\text{Pb}$

<table>
<thead>
<tr>
<th>Particle</th>
<th>EM coupling</th>
<th>Weak-Vector coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>up-quark</td>
<td>$+2/3$</td>
<td>$+1 - 4\sin^2\theta_w(+2/3) \approx +1/3$</td>
</tr>
<tr>
<td>down-quark</td>
<td>$-1/3$</td>
<td>$-1 - 4\sin^2\theta_w(-1/3) \approx -2/3$</td>
</tr>
<tr>
<td>proton</td>
<td>$+1$</td>
<td>$+1 - 4\sin^2\theta_w \approx 0$</td>
</tr>
<tr>
<td>neutron</td>
<td>$0$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

$g_V^f = 2T_z^f - 4\sin^2\theta_w Q_f^f$, \hspace{1cm} \sin^2\theta_w \approx 0.231 \approx 1/4$
High Densities in Earth
[Danielewicz, Lacey, and Lynch – Science 298, 1592 (2002)]

Nuclear Collisions: Constraints and Predictions
- Sole earthly tool available to compress nuclear matter
- Compressions up to several (five) times nuclear saturation density
- Imprint of the EoS left in the flow and fragment distribution

FSUGold provides a reliable extrapolation to high density ...
Neutron Skin and Neutron-Star Radii

**Question:** Is there a correlation between the neutron skin of $^{208}\text{Pb}$ and the radius of a “canonical” $1.4M_\odot$ neutron star?

**Answer:** Probably yes! Same pressure that pushes neutrons out in $^{208}\text{Pb}$ pushes neutrons out in a neutron star.

- Isolated radio-quiet neutron stars already discovered
- Find good candidates for mass-radius measurement

**Interesting Correlation:**
The neutron skin of $^{208}\text{Pb}$ depends on the EOS below N.M. saturation density, while the radius of the neutron star is also sensitive to the high-density EOS. “The thinner the skin of $^{208}\text{Pb}$, the smaller the radius of the star”

Large neutron skin together with a small neutron-star radius, could provide strong signature in favor of a phase transition ...
Maximum (Limiting) Neutron-Star Mass

- Maximum mass determined by high-density behavior of EOS ($\zeta$)
- Radius of low-mass stars determined by symmetry energy ($\Lambda_v$)

At present both parameters are poorly constrained!
However, situation could improve very rapidly ...

- Find a single “heavy” neutron star (PSR J0751?)
- Measure the neutron radius of $^{208}$Pb (JLAB?)
Neutron Star Composition

The composition of non-exotic stars is controlled by the density dependence of the symmetry energy!

- Symmetry energy imposes a penalty for violating \((N=Z)\) balance
- The stiffer the symmetry energy, the higher the price
- The stiffer the symmetry energy, the higher the proton fraction
Electron Fraction and Neutron-Star Cooling

Enhanced (direct URCA) cooling of non-exotic stars?

- Core-collapse Supernovae generates proto-neutron star ($T_{\text{core}} \approx 10^{12}\text{K}$)
- Direct URCA process cools down the star until ($T_{\text{core}} \approx 10^{9}\text{K}$)
- Depending on the EoS direct URCA may continue
  - Is $Y_p$ large enough to conserve momentum?
- Best case for DUrca (soft-stiff):
  - Soft EoS for symmetric matter $\rightarrow$ large $\rho_c$
  - Stiff symmetry energy $\rightarrow$ large $Y_p$

Direct URCA process:

a) $n \rightarrow p + e^- + \bar{\nu}_e$

b) $p + e^- \rightarrow n + \nu_e$

may continue cooling the neutron star.

FSUGold predicts that the pulsar in 3C58 does NOT need to be an exotic (quark) star.
High Densities in Heaven

Neutron Stars: Constraints and Predictions

- Sole heavenly tool available to compress nuclear matter
- Compression up to several (ten?) times nuclear saturation density
- Imprint of the EoS left in limiting mass, radius, cooling history, ...

<table>
<thead>
<tr>
<th>Model</th>
<th>$k^0_F$ (fm$^{-1}$)</th>
<th>$\epsilon_0$ (MeV)</th>
<th>$K$ (MeV)</th>
<th>$J$ (MeV)</th>
<th>$L$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL3</td>
<td>1.30</td>
<td>-16.2</td>
<td>271</td>
<td>37.4</td>
<td>118.5</td>
</tr>
<tr>
<td>FSUGold</td>
<td>1.30</td>
<td>-16.3</td>
<td>230</td>
<td>32.6</td>
<td>60.5</td>
</tr>
</tbody>
</table>

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</tr>
</thead>
<tbody>
<tr>
<td>$\rho_c$ (fm$^{-3}$)</td>
<td>0.052</td>
<td>0.085</td>
<td>0.051</td>
<td>0.076</td>
</tr>
<tr>
<td>$R$ (km)</td>
<td>15.05</td>
<td>14.18</td>
<td>13.80</td>
<td>12.66</td>
</tr>
<tr>
<td>$M_{\text{max}}(M_{\odot})$</td>
<td>2.78</td>
<td>2.75</td>
<td>1.80</td>
<td>1.72</td>
</tr>
<tr>
<td>$\rho_{\text{Urca}}$ (fm$^{-3}$)</td>
<td>0.21</td>
<td>0.51</td>
<td>0.22</td>
<td>0.47</td>
</tr>
<tr>
<td>$M_{\text{Urca}}(M_{\odot})$</td>
<td>0.84</td>
<td>2.64</td>
<td>0.74</td>
<td>1.30</td>
</tr>
<tr>
<td>$\Delta M_{\text{Urca}}$</td>
<td>0.38</td>
<td>0.00</td>
<td>0.59</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Some Questions and Answers (FSUGold):

- Is the pulsar in 3C58 an exotic star?
  Not necessarily if $M_{\ast} > 1.3 \, M_{\odot}$
- Is the limiting mass of a neutron star $M_{\text{max}} \approx 1.72 \, M_{\odot}$?
  Report suggests $M(PSRJ0751 + 1807) = 2.1^{+0.4}_{-0.5}$

Fascinating times ahead!