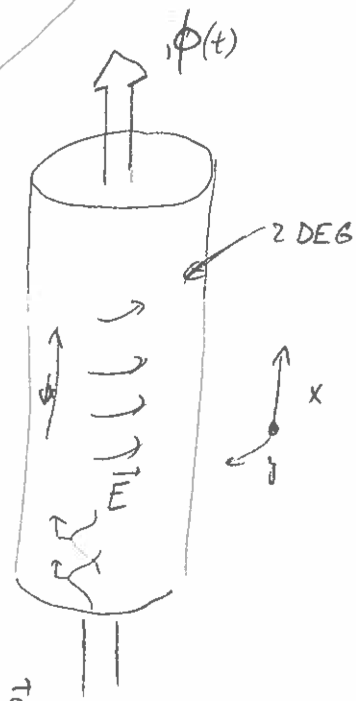
Quantization of σ_{xy} and the Laughlin argument.

- When an integer # of LL is filled we have an insulator, i.e. $\sigma_{xx} = \sigma_{yy} = 0$ "Quantum Hall insulator"
- However $\sigma_{xy} = \frac{e^2}{h} n \qquad n \in \mathbb{Z}$



Thread flux $\Phi(t)$

Read Ch. 3 textbook for a direct calc of σ_{xy}

$$\delta \vec{A}(t) = \left(0, \frac{\Phi(t)}{L_y}, 0\right)$$

$$\oint dl\, \delta \vec{A}(t) = \Phi(t)$$

$\Phi(t)$ will generate Faraday el. field

$$E_y = \frac{1}{c}\frac{d\phi}{dt} \qquad \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

Current:

$$j_x = \sigma_{xy} E_y = \frac{1}{c}\sigma_{xy}\frac{d\phi}{dt}$$

Transferred charge

$$\boxed{Q = \int_0^T j_x\, dt = \frac{1}{c}\sigma_{xy}\left[\Phi(T) - \Phi(0)\right] = \frac{1}{c}\sigma_{xy}\Delta\phi}$$

Chose $\Delta\phi = \Phi_0 = \frac{hc}{e}$ Laughlin argument essence

$\rightarrow$ this is INVISIBLE TO ELECTRONS (2π Aharonov Bohm phase)

- can be removed from H by a gauge transf.

$\Rightarrow$ the H is identical $\quad \psi \rightarrow e^{i 2\pi \frac{y}{L_y}} \psi$

$$H(t) = \frac{1}{2m}\left[p_x^2 + \hbar^2\left(-i\partial_y - \frac{e\hbar}{c}\delta A_y(t)\right)^2\right]$$

$$\delta A_y = \frac{\phi(t)}{L_y}$$

$$\Phi_0 = \frac{hc}{e}$$

$$H(0) = \frac{1}{2m}\left[p_x^2 + \hbar^2(-i\partial_y)^2\right]$$

$$H(T) = \frac{1}{2m}\left[p_x^2 + \hbar^2\left(-i\partial_y - \frac{e\hbar}{c}\frac{\phi_0}{L_y}\right)^2\right]$$

$$\xrightarrow{\text{gauge}} \frac{1}{2m}\left[p_x^2 + \hbar^2\left(-i\partial_y + \frac{2\pi}{L_y} - \frac{2\pi}{L_y}\right)^2\right] = H(0)$$

The ground state has changed AT MOST by transferring an integer # of electrons n from one end to another, $Q = ne$

$$ne = \frac{1}{c}\sigma_{xy}\frac{hc}{e} \Rightarrow \boxed{\sigma_{xy} = \frac{e^2}{h}n} \quad n \in \mathbb{Z}$$

FRACTIONAL QUANTUM HALL EFFECT (FQHE)

exp: $\sigma_{xy} = \frac{e^2}{h}\left(\frac{n}{m}\right)$ $n, m \in \mathbb{Z}$ - high purity samples, low T, high B.

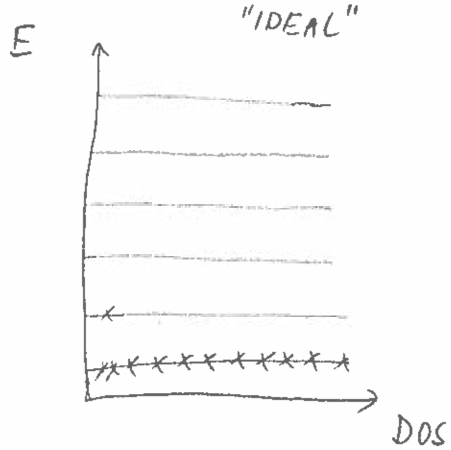
Laughlin: $Q = \frac{1}{c}\sigma_{xy}\frac{hc}{e} = \frac{n}{m}e$

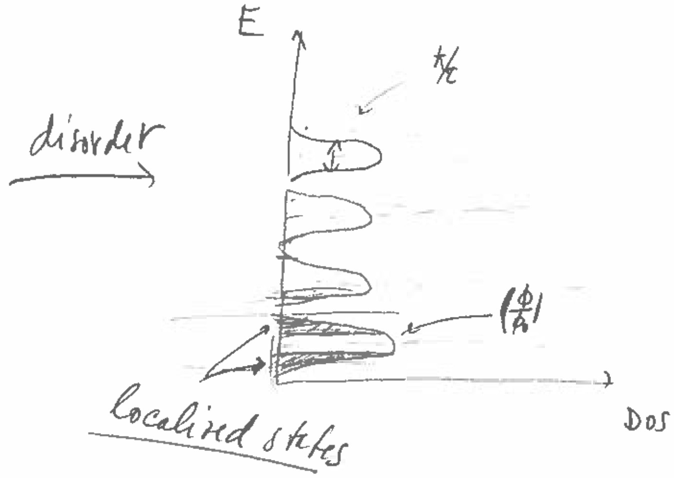
also fractional exchange statistics.

FQHE has excitations with fractional charge, e.g. $\frac{e}{3}$

$\Rightarrow$ interaction effect, cannot be understood within the indep. electron picture.

Real IQHE

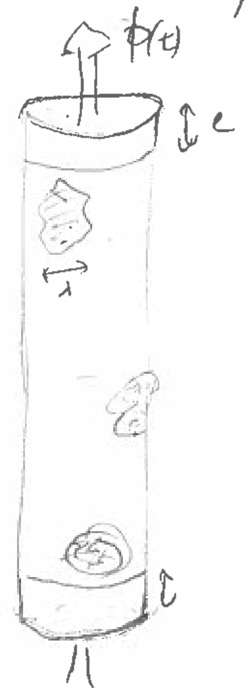




do not contribute to transport.

$$D(E) = \left(\frac{\Phi}{\Phi_0}\right) \sum_n \delta(E - \varepsilon_n) \; ,$$

$$\varepsilon_n = \hbar\omega_c\left(n + \tfrac{1}{2}\right)$$



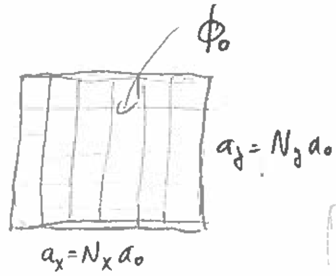
Laughlin argument goes through for localized states.

$\Rightarrow$ leads to QH plateaus of finite width in B.

TKNN invariant and the Chern insulator

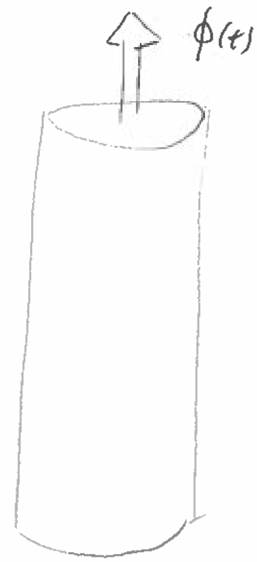
Thouless, Kohmoto, Nightingale & den Nijs [PRL, 49, 405 (1982)]

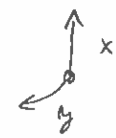
Consider 2D solid in commensurate B-field



$\sigma_{xy} = \frac{e^2}{h} n$

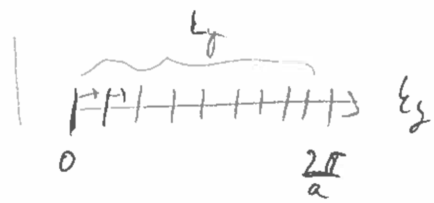
$\boxed{n = \text{Chern } \#}$





P.b.c: $k_x = \frac{2\pi}{L_x} n_x \qquad (L_x \to \infty)$

$$k_y^{n_y} = \frac{2\pi}{L_y}\left(n_y + \frac{\phi(t)}{\phi_0}\right), \quad n_y = 1 \dots L_y$$



$$H = \frac{1}{2m}\left[p_x^2 + \hbar^2\Big(-i\partial_y \underbrace{- \frac{1}{L_y}\frac{\phi}{\phi_0}}_{-\frac{e}{\hbar c}A_y(x)}\Big)^2\right] + V(x,y)$$

$$\psi(x,y) \to e^{2\pi i \frac{y}{L_y}\frac{\phi}{\phi_0}}\psi(x,y)$$

As $\phi(t)$ varies $0 \to \phi_0$ this constitutes a Thouless charge pump, $\delta Q = en$

Berry curvature

$$n = \frac{1}{2\pi}\sum_{n_y}\int_0^T dt \int_{-\pi/a}^{\pi/a} dk_x \; \mathcal{F}_{k_x t}\left(k_x, k_y^{n_y}(\phi(t))\right)$$

$$= \frac{1}{2\pi}\sum_{n_y}\int_0^{\phi_0} d\phi \int dk_x \; \mathcal{F}_{k_x\phi}\left(k_x, k_y^{n_y}(\phi)\right)$$

$$= \frac{1}{2\pi}\int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dk_x \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} dk_y \; \mathcal{F}_{k_x k_y}(k_x, k_y)$$

$\leftarrow$ Chern #.

$\phi(t) = \frac{t}{T}\phi_0$

$d\phi = \frac{\phi_0}{T} dt$

$k_y = \frac{2\pi}{L_y}\left(n_y + \frac{\phi}{\phi_0}\right)$

$dk_y = \frac{2\pi}{L_y}\frac{1}{\phi_0} d\phi = \frac{2\pi}{L_y}\frac{dt}{T}$