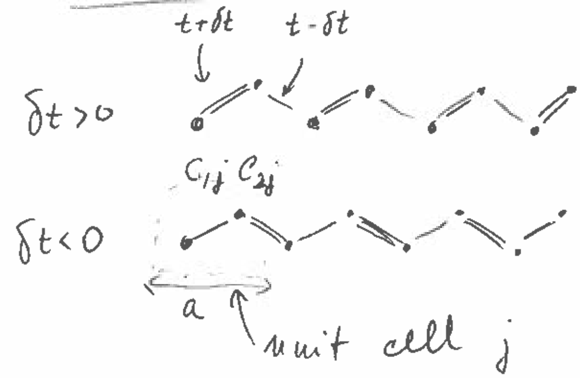
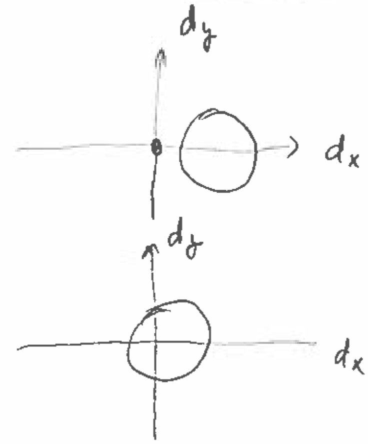
EXAMPLE: Polyacetylene and the Su-Schrieffer-Heeger model





$$H = \sum_j \left[(t+\delta t)\, c^\dagger_{1j} c_{2j} + (t-\delta t)\, c^\dagger_{1j+1} c_{2j} + h.c. \right]$$

$$c_{aj} = \frac{1}{\sqrt{N}} \sum_k e^{ikj} c_{ak} \qquad a = 1,2 \qquad \text{(disregard electron spin for now)}$$

$$H = \sum_k H_{ab}(k)\, c^\dagger_{ak} c_{bk} \qquad H(k) = \vec{d}(k)\cdot\vec{\sigma}$$

$$d_x(k) = \overbrace{(t+\delta t)}^{t_1} + \overbrace{(t-\delta t)}^{t_2} \cos ka \qquad t_1 \equiv t+\delta t$$

$$d_y(k) = (t-\delta t)\sin ka \qquad t_2 \equiv t-\delta t$$

$$d_z(k) = 0$$

Spectrum:

$$E_k = \pm|\vec{d}| = \pm\sqrt{\vec{d}\cdot\vec{d}} = \pm\sqrt{(t_1+t_2\cos k)^2 + t_2^2 \sin^2 k}$$

$$= \pm\sqrt{t_1^2+t_2^2+2t_1t_2\cos k} \;\circleddash$$

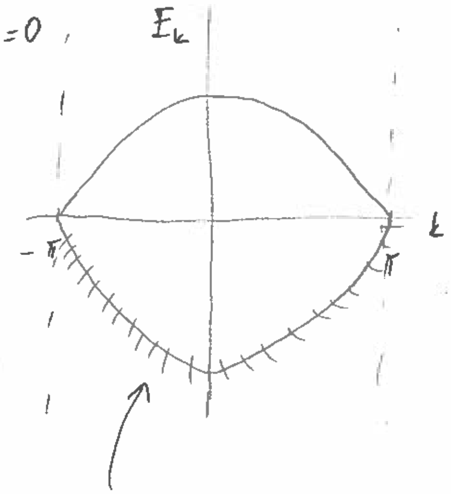
$$= \cancel{\pm\sqrt{2(t^2+\delta t^2)+2(t^2-\delta t^2)\cos k}}$$

Special case $\delta t = 0 \Rightarrow t_1 = t_2 = t$

$$E_k = \pm t\sqrt{(1+\cos k)^2 + \sin^2 k} = \pm t\sqrt{2(1+\cos k)} = \pm 2t\left|\cos\frac{k}{2}\right|$$

$$\circleddash\; \pm\sqrt{(t_1-t_2)^2 + 2t_1t_2(\cos k + 1)} = \pm\sqrt{(2\delta t)^2 + 4(t^2-\delta t^2)\cos^2\frac{k}{2}}$$

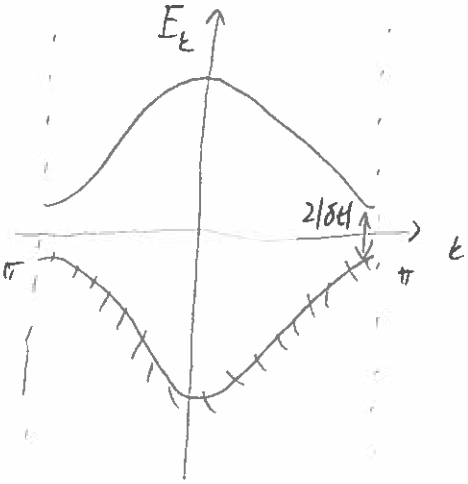
$\delta t = 0$,



At half filling

$\to$ metal

$\delta t \neq 0$



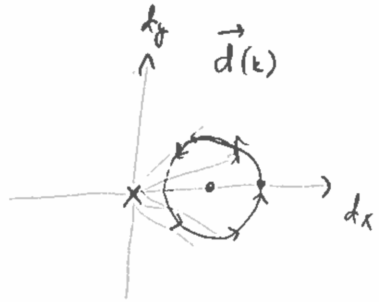
insulator

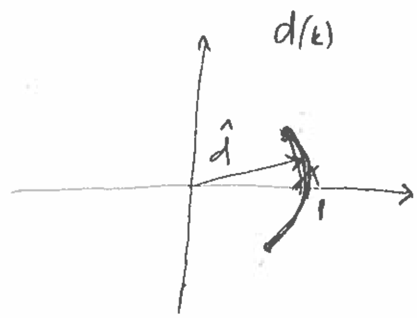
- $\underline{\delta t = 0}$ is a topological phase transition between two distinct insulators for $\delta t > 0$, $\delta t < 0$.

Consider the Berry phase

$d_z = 0 \quad \Rightarrow \quad \vec{d}$ is in x-y plane

$\delta t > 0$
$(t_1 > t_2)$





solid angle

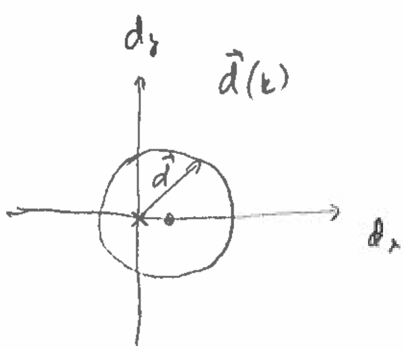
$\downarrow$

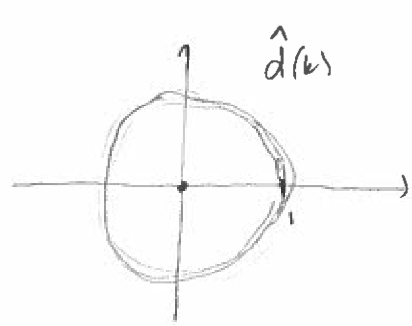
$\Omega = 0$

$\gamma_c = 0$

$P = 0$

$\delta t < 0$
$(t_1 < t_2)$





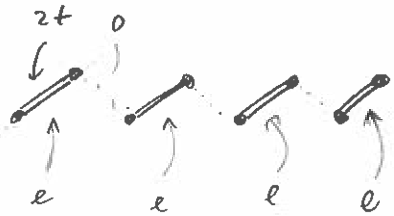
$\Omega = 2\pi$

$\gamma_c = \frac{1}{2}\Omega = \pi$

$P = \frac{e}{2\pi}\gamma_c = \frac{e}{2}$

Consider an extreme dimerization limit $\delta t = \pm t$:

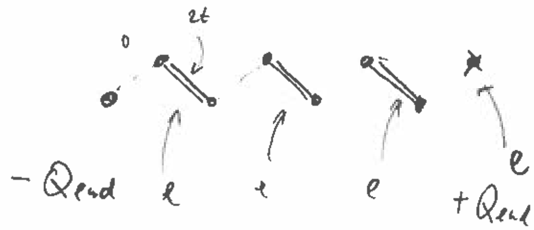
$\delta t = + t$



$\frac{1}{2}e$ per site ✓

$P = 0$

$\delta t = - t$



$\frac{1}{2}e$ per site in the bulk

$P = \frac{1}{2}e$

- <u>Chiral symmetry</u>
  - the sharp distinction between the two phases only exists when $d_z = 0$.

    For $d_z \neq 0$ one can continuously extrapolate between $\delta t < 0$ and $\delta t > 0$, and P can be arbitrary (see HWK # 2).
- $d_z = 0$ situation is protected by chiral symmetry, $\sigma_z H \sigma_z = - H$ (Check!)

$\Rightarrow$ SYMMETRY - PROTECTED TOPOLOGICAL ORDER

Real polyacetylene has inversion symmetry,

$$\sigma_x H(k) \sigma_x = H(-k) \quad \text{PLUS time reversal} \quad H^*(k) = H(-k)$$

which also works.

Domain wall states and the Jackiw - Rebbi model (1981)

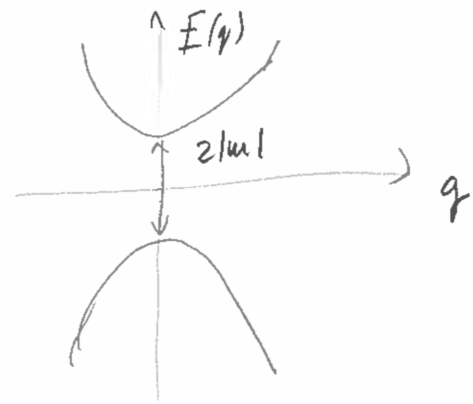
Low-energy continuum model

consider $|\delta t| \ll t$ and $k = \pi + q$, small $|q| \ll \pi$

$$d_x \simeq 2\delta t \equiv m$$
$$d_y \simeq (at)q \equiv vq \qquad \text{(restoring the latt. const.)}$$

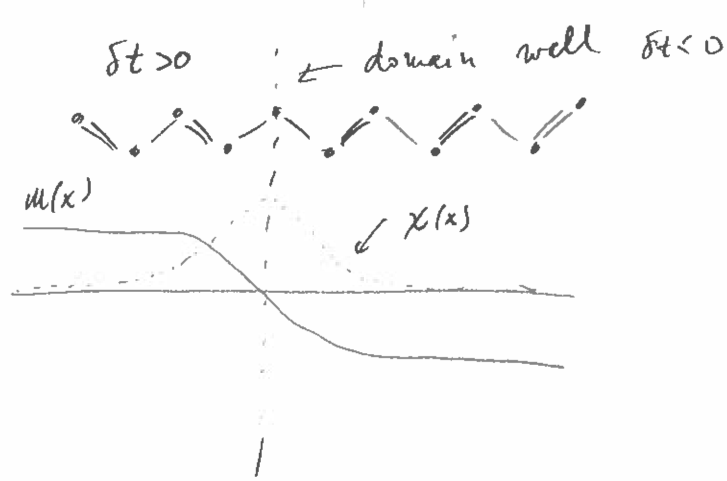
$$H_{eff} = \sigma_x m + \sigma_y v q \qquad q \to -i\partial_x$$

$$E(q) = \sqrt{v^2 q^2 + m^2} \quad \Rightarrow \text{"massive Dirac Hamiltonian in 1D"}$$



$$H_{eff} = \sigma_x m(x) + \sigma_y v(-i\partial_x)$$

Jackiw - Rebbi zero mode



$$H_{eff}\Psi = 0$$

$$H_{eff} = \begin{pmatrix} 0 & m - v\partial_x \\ m + v\partial_x & 0 \end{pmatrix}$$

$$\Psi_1 = \begin{pmatrix} 0 \\ \chi \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} \xi \\ 0 \end{pmatrix}$$

$$[m(x) - v\partial_x]\chi = 0 \qquad \chi(x) = A\, e^{+\frac{1}{v}\int_0^x m(x')dx'}$$

$\Rightarrow$ single localized zero mode attached to the domain wall

Protected by the chiral (inversion) symmetry.

Majorana modes in SC.