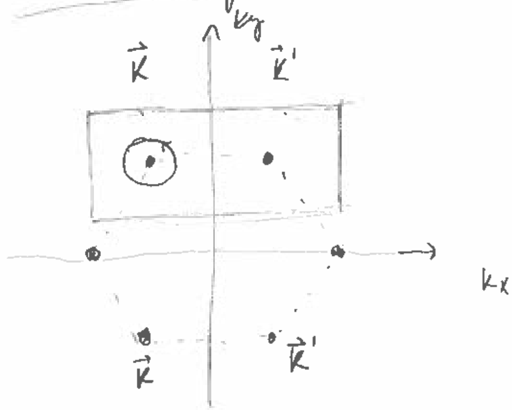
LECTURE 22

TOPOLOGY IN GAPLESS SYSTEMS: DIRAC & WEYL SEMIMETALS

(I) Edge states in gapless graphene

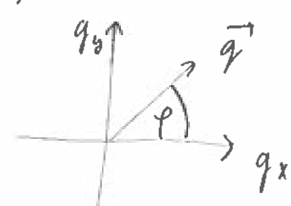


$\vec{K}$: $\mathcal{H}_{eff}(q) = v_F(\sigma_x q_x + \sigma_y q_y)$

$\vec{K}'$: $\mathcal{H}'_{eff}(q) = v_F(-\sigma_x q_x + \sigma_y q_y)$

Consider a small circle around $\vec{K}$ and compute the Berry phase integral along that circle: $\mathcal{L} = \oint \vec{\mathcal{A}} \cdot d\vec{l}$

$$\mathcal{H} = \vec{d} \cdot \vec{\sigma} \qquad \vec{d} = v_F q(\cos\varphi, \sin\varphi, 0)$$



Problem set # 2

$$\vec{d} = d(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

$$E = \pm|\vec{d}|, \qquad |\phi_-\rangle = \begin{pmatrix} \sin\frac{\theta}{2} e^{-i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

Only non-zero component of Berry curvature

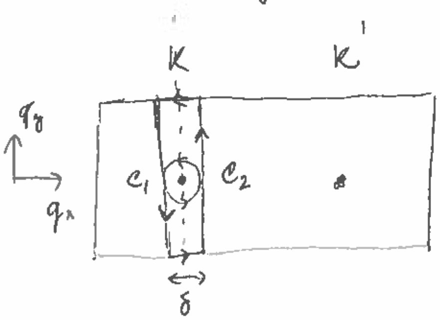
$$\mathcal{F}_{\theta\varphi} = \frac{1}{2}\sin\theta$$

$$A_\varphi = i\langle\phi|\tfrac{\partial}{\partial\varphi}|\phi\rangle = \sin^2\tfrac{\theta}{2} \quad , \quad A_d = A_\theta = 0$$

For our case $\theta = \frac{\pi}{2}$ so $A_\varphi = \sin^2\frac{\pi}{4} = \frac{1}{2}$

$$\boxed{\Omega = \oint_c \vec{A}\cdot d\vec{l} = \int_0^{2\pi} A_\varphi\, d\varphi = \tfrac{1}{2}\cdot 2\pi = \pi}$$

- Now deform the circle: since Ω is indep. of circle radius all the contribution must come from the origin.



As $\delta \to 0$ all the contribution to Ω must come from the vertical segments:

$$\Omega = \int_{c_1} \vec{A}\cdot d\vec{l} + \int_{c_2} \vec{A}\cdot d\vec{l} = \pi$$

Mental leap: View $\mathcal{H}(q_x, q_y) \equiv \mathcal{H}_{q_x}(q_y)$ as a collection of 1D Hamiltonians for each value of q_x

$$\mathcal{H}_{q_x}(q_y) = v q_x \sigma_x + v q_y \sigma_y \equiv m_x \sigma_x + v q_y \sigma_y \qquad m_x = v q_x$$

1D Dirac Hamiltonian with mass $m_x = v q_x$

$\rightarrow$ gapped 1D Hamiltonian for all $q_x \neq 0$.

- From our discussion of Polyacetylene we know that
$$\Omega_c = \int_{1D\,Bz} \mathcal{A}\cdot dq_y = \begin{cases} 0 \\ \pi \end{cases} \quad \text{when} \quad d_z = 0.$$

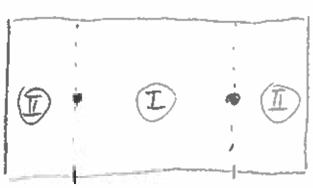
Also, physically $P = \frac{e}{2\pi}\Omega_c$

So: $$\Omega = \overset{0,\pi}{\Omega_{c_1}} + \overset{0,\pi}{\Omega_{c_2}} = \pi$$

Two possibilities: $\Omega_{c_1} = 0,\ \Omega_{c_2} = \pi$

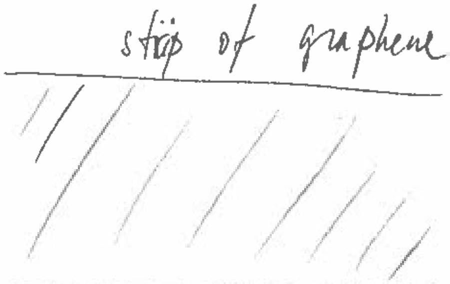
or $\Omega_{c_1} = \pi,\ \Omega_{c_2} = 0$

- Conclusion:

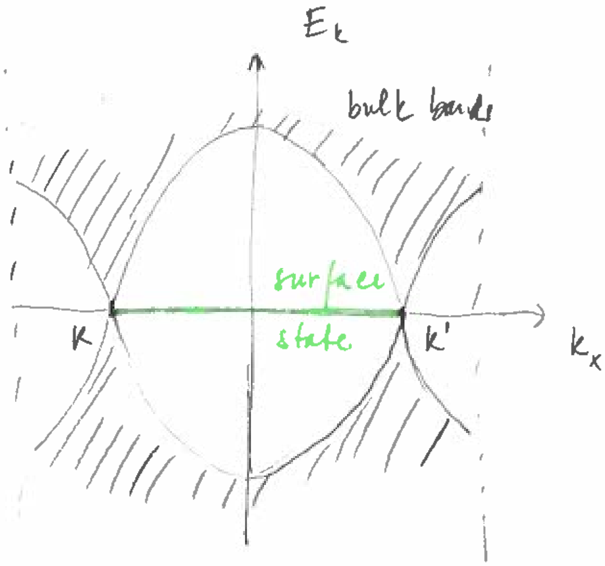


A 1D system associated with region I or region II is topological (in the same way as dimerized polyacetylene)

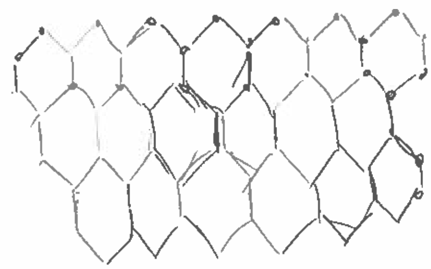
- Physical consequence:



A strip of graphene, cut such that there is a non-zero projection of $\vec{K}-\vec{K}'$ vector onto the edge, has a gapless edge state for momenta k_x between the nodes $\vec{K}$ and $\vec{K}'$.



"Flat surface band" exists in the "zigzag" edge geometry



"armchair edge"
no edge state

- In 2D this physics also occurs in d-wave superconductors (high-T_c cuprates) which also have Dirac fermions at low energies. In this case edge modes are Majorana fermions.