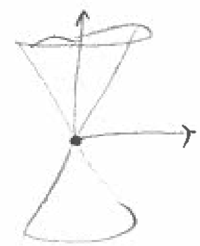
LECTURE IV

Surface states in STIs

2D massless Dirac Hamiltonian:

$E(\vec{k}) = \pm v\sqrt{k_x^2 + k_y^2}$



$$\mathcal{H}^{surf}(\vec{k}) = v\left(-s_x k_y + s_y k_x\right) = v\,\vec{s}\cdot(\hat{z}\times\vec{k})$$

$\mathcal{T}$-invariant: $\quad s_y \mathcal{H}^*(\vec{k})\, s_y = \mathcal{H}(-\vec{k})$ ✓

- The state is protected by $\mathcal{T}$; to open a gap one needs
$$\delta\mathcal{H} = s_z m$$
but this is odd under $\mathcal{T}$.

- Backscattering is prohibited: $\quad [\theta, V] = 0$
$$\langle \psi(\vec{k}) | V | \psi(-\vec{k}) \rangle = 0$$
(proof analogous to 2D edge state)

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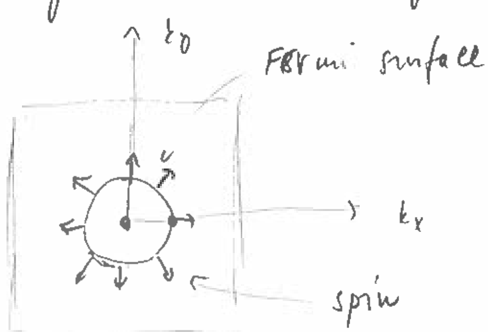


But scattering $\vec{k} \to -\vec{k} + \vec{\delta}$ is allowed

$\Rightarrow$ transport is not ballistic but states cannot localize

Caveat: inelastic scattering (e.g. by phonons, e-e interaction) can lead to backscattering but this effect is usually weak; $l_I \simeq \mu m$ (inelastic mean-free path)

Theory of localization (Anderson, 1958)

Free electrons + disorder:

$$H = \frac{\hbar^2 \nabla^2}{2m} + V(\vec{r})$$

random

$$\langle V(\vec{r}) V(\vec{r}') \rangle = U^2 \delta(\vec{r} - \vec{r}')$$

$$\langle V(\vec{r}) \rangle = 0$$

– "δ-correlated" random potential with amplitude U.

1D	All states localized for arbitrarily weak U.
2D	Marginal case: Low-E states localized but loc. length exponentially large
3D	Low-E states localized when $U > U_c$.

"Weak localization", quantum effect ⇒ disordered systems in 1D, 2D are insulating!

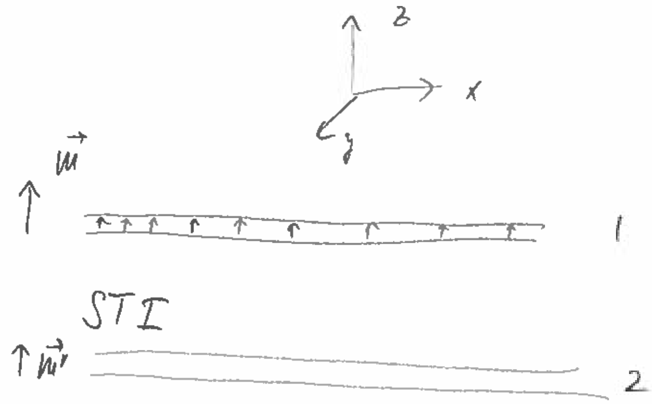
For Dirac Hamiltonian, with V T-invariant

$$H_D = i v \vec{\sigma} \cdot \vec{\nabla} + V(\vec{r})$$

states are EXTENDED in all dimensions.

No weak localization!

Magnetized surface states



1: $\mathcal{H}(\vec{k}) = S_x k_x + S_y k_y - \vec{m}\cdot\vec{S}$

$$= S_x(k_x - m_x) + S_y(k_y - m_y) - S_z m_z$$

$$\boxed{E_k = \pm\sqrt{(k_x - m_x)^2 + (k_y - m_y)^2 + m_z^2}}$$

[m_x, m_y shift the surface Dirac cone
m_z opens a gap]

$\Rightarrow$ Breaking of $\mathcal{T}$ is a necessary but not sufficient condition for removing the gapless state

Focus on m_z

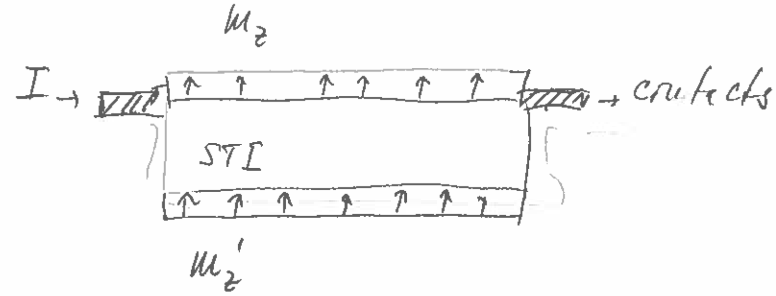
We know from our discussion of graphene that a gapped Dirac fermion has non-zero σ_{xy}

Namely $\sigma_{xy} = \frac{e^2}{h} n$ $n = \frac{1}{2}\,\text{sign}(m_z)$ for Dirac fermion

Expect $\boxed{\sigma_{xy} = \frac{e^2}{2h}\,\text{sign}(m_z)}$ per surface

Fractional QHE in a non-interacting system

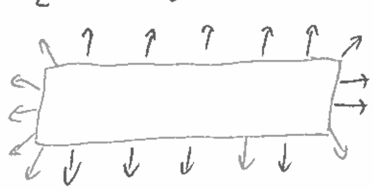
This is a special property of a STI.
However, cannot be easily measured in transport because of contribution from the other surface.



Two surfaces:

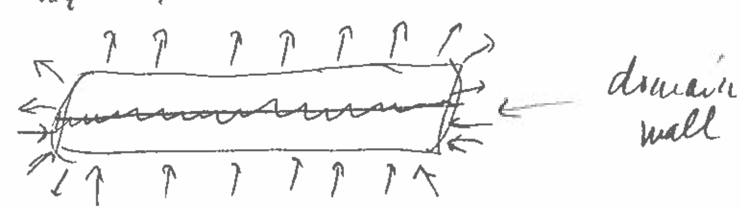
$$\sigma_{xy}^{tot} = \frac{e^2}{2h}\left[\mathrm{sign}(m_z) + \mathrm{sign}(m_z')\right] = \begin{cases} 0, & m_z = -m_z' \\ \pm\frac{e^2}{h}, & m_z = m_z' \end{cases}$$

$m_z = -m_z'$



no edge state
expect $\sigma_{xy} = 0$

$m_z = m_z'$



a single edge state,
expect $\sigma_{xy} = \pm\frac{e^2}{h}$

Q: Can one measure the fractional σ_{xy}?

A: Yes, in an experiment that probes a single surface, E.g. optical Kerr effect

Change in reflected light polarization due to σ_{xy}