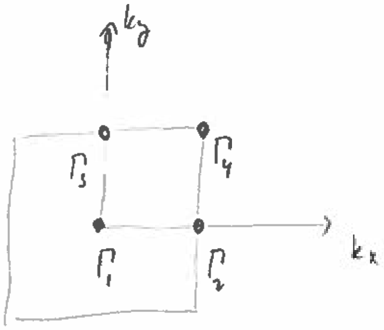
LECTURE 13

$\mathbb{Z}_2$ invariant recap

- For inversion symmetric 2D insulators we showed that

$$(-1)^{\nu} = \prod_{i=1}^{4} \delta_i \, , \qquad \text{with} \quad \delta_i = \prod_{m=1}^{N} \xi_{2m}(\vec{\Gamma}_i)$$

parity eigenvalues



$\vec{\Gamma}_i$ – TRIM (time-reversal invariant momenta)

- Often one uses adiabatic continuity to classify systems <u>without</u> inversion symmetry.

$$H = \underset{\substack{\uparrow \\ \mathcal{P}\text{-symm.}}}{H_0} + \underset{\substack{\uparrow \\ \mathcal{P}\text{-breaking}}}{H_1} \quad \longrightarrow \quad H(\lambda) = H_0 + \lambda H_1$$

– find ν for H_0

– study spectrum of $H(\lambda)$: if gap does not close as $\lambda: 0 \to 1$ then H is in the same phase

• EXAMPLE: $\mathbb{Z}_2$ invariant for the Kane-Mele model of Graphene

$$H_{KM} = \sum_{\vec{k}} \overbrace{\left[\mathcal{H}_0^{\alpha\beta}(\vec{k}) + \delta\mathcal{H}_{KM}^{\alpha\beta}(\vec{k})\right]}^{\mathcal{H}^{\alpha\beta}(\vec{k})} c^{+}_{k\alpha} c_{k\alpha}$$

$$\mathcal{H}_0(\vec{k}) = \vec{d}(\vec{k})\cdot\vec{\sigma} \qquad d_x(\vec{k}) = -t\sum_{p=1}^{3}\cos(\vec{k}\cdot\vec{\delta}_p)$$

$$d_y(\vec{k}) = -t\sum_{p=1}^{3}\sin(\vec{k}\cdot\vec{\delta}_p)$$

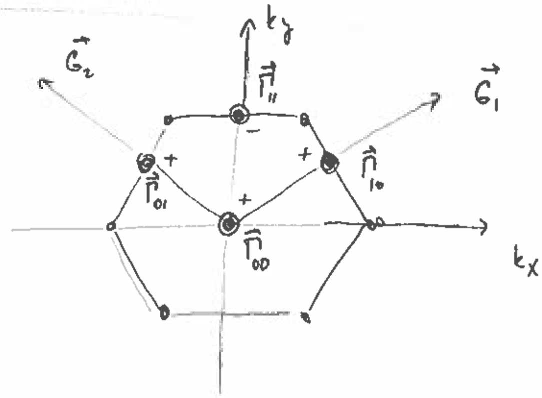
$$d_z(\vec{k}) = 0$$

$$\delta\mathcal{H}_{KM}(\vec{k}) = \lambda_{so}\,\sigma_z S_z \sum_{p<p'=1}^{3} S_{pp'} \sin\vec{k}\cdot(\vec{\delta}_p - \vec{\delta}_{p'}) \,, \qquad S_{pp'} = \pm \text{ for } p' = (p\pm 1)\,\mathrm{mod}$$

H_{KM} obeys both $\mathcal{T}$ and $\mathcal{I}$:

$\mathcal{T}$: $\quad \Theta\,\mathcal{H}(\vec{k})\,\Theta^{-1} = \mathcal{H}(-\vec{k})\,, \quad \Theta = i s_y K$

$\mathcal{I}$: $\quad P\,\mathcal{H}(\vec{k})\,P^{-1} = \mathcal{H}(-\vec{k})\,, \quad P = \sigma_x$



It holds that:

TRIM: $\vec{\Gamma}_{n_1 n_2} = \frac{1}{2}(n_1\vec{G}_1 + n_2\vec{G}_2)\,, \quad n_j = 0,1$

$$\boxed{\mathcal{H}(\vec{\Gamma}_i) = d_x(\vec{\Gamma}_i)\,P}$$

$$\left[d_y(\vec{\Gamma}_i) = \delta\mathcal{H}_{KM}(\vec{\Gamma}_i) = 0\right]$$

- these terms are prohibited by symmetry, e.g. $P\delta\mathcal{H}_{KM}(\vec{\Gamma})P^{-1} = \delta\mathcal{H}_{KM}$

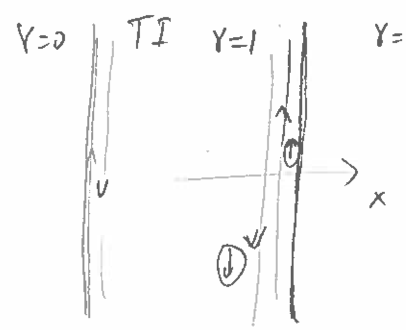
Since eigenvalues of $\mathcal{H}(\vec{\Gamma})$ for the occupied states are negative it follows that

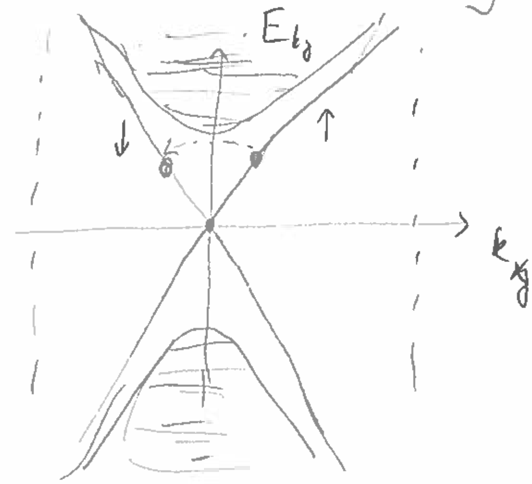
$$\boxed{\delta_i = -\,\mathrm{sgn}\left[d_x(\vec{\Gamma}_i)\right]}$$

$$\left.\begin{array}{l}\delta_{00} = \delta_{01} = \delta_{10} = +1 \\ \delta_{11} = -1\end{array}\right\} \Rightarrow \nu = 1 \qquad \boxed{H_{KM} \text{ is a 2D TI}}$$

- <u>Properties of the edge states</u>

[When $\gamma = 1$ a pair of topologically protected, counter-propagating, spin filtered edge states exists.]



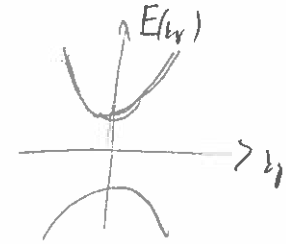


(I) The edge states are protected by $\mathcal{T}$

(II) Elastic backscattering is prohibited $\Rightarrow$ edge states cannot be localized.

(I) Low-E theory

$$\mathcal{H}^{edge}(k_y) = v\, s_z k_y$$

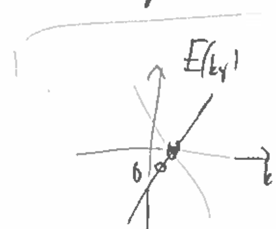


$$\mathcal{T}: \quad s_y \mathcal{H}^*(\vec{k}) s_y = \mathcal{H}(-\vec{k}) \quad \checkmark$$

To open a gap, we must add

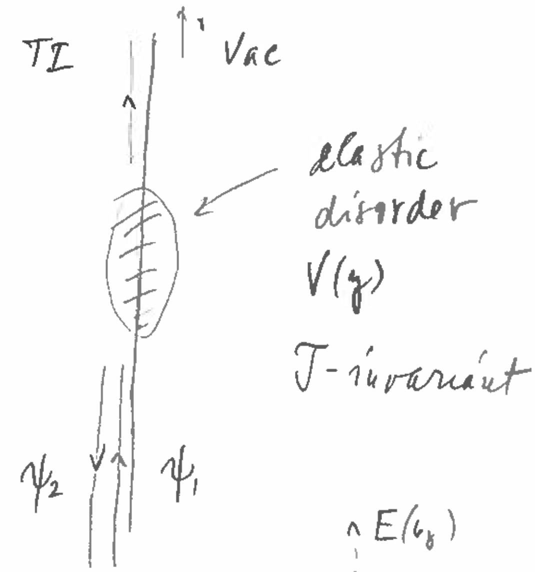
$$\delta\mathcal{H} = m_x s_x + m_y s_y, \qquad E(k_y) = \pm\sqrt{(v k_y)^2 + m_x^2 + m_y^2}$$

but $s_y\, \delta\mathcal{H}^* s_y = -\delta\mathcal{H}$ $\times$ breaks $\mathcal{T}$.



$$\delta\mathcal{H}' = m_z s_z \qquad E'(k_y) = \pm|v k_z + m_z|$$

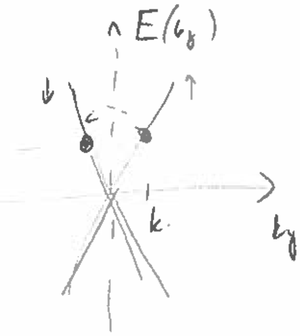
(II) Backscattering



$$\mathcal{H}^{edge} = v\, s_z(-i\partial_y) + V(y)$$

ψ_1 : incoming

ψ_2 : reflected



$|\psi_2\rangle = \Theta|\psi_1\rangle$ $\qquad \Theta$

$\Theta|\psi_2\rangle = -|\psi_1\rangle$ $\qquad \Theta^2 = -1$

Interested in reflection (backscattering) amplitude

$$\begin{aligned}
R = \langle\psi_1|V|\psi_2\rangle &= \langle\psi_1|V\Theta|\psi_1\rangle = \\
&= \langle\psi_1|\Theta V|\psi_1\rangle \\
&= -\langle\Theta\psi_2|\Theta V\psi_1\rangle \\
&= -\langle V\psi_1|\psi_2\rangle \\
&= -\langle\psi_1|V|\psi_2\rangle = -R
\end{aligned}$$

$[V,\Theta]=0$

$\langle\Theta a|\Theta b\rangle = \langle a|b\rangle^* = \langle b|a\rangle$

V - hermitian

$\boxed{R = 0}$ Backscattering by T-invariant elastic disorder is prohibited.

$\Rightarrow$ ballistic transport along the edge.

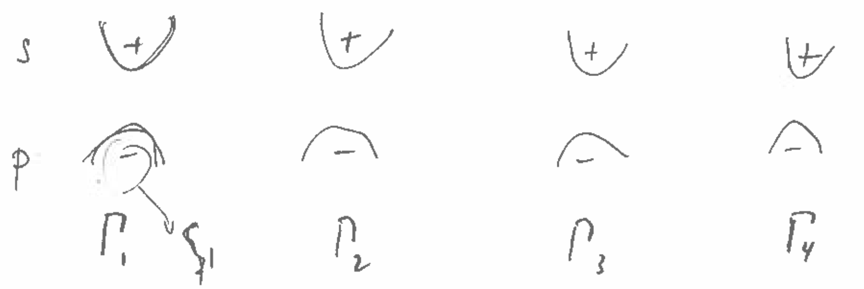
HgTe : The first experimental example of a TI.

[Th: Bernevig, Hughes & Zhang, Science $\underline{314}$ 1757 (2006) | exp: König et al, Science $\underline{318}$, 766 (2007)

HgTe is a II-VI semiconductor

crystalizes in zincblende structure (FCC with 2-point basis)

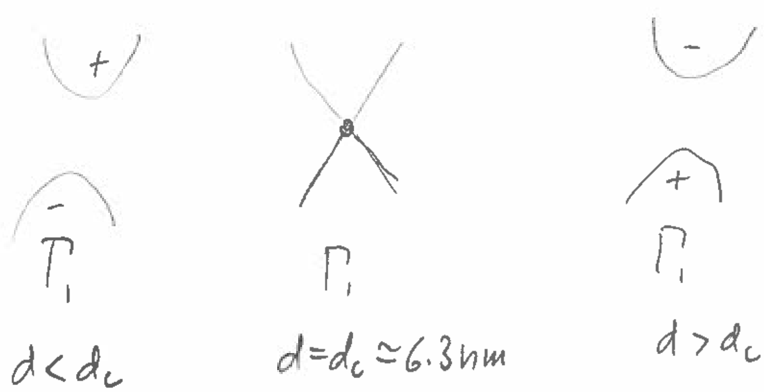
(ZnS, CuF, MnSe) dozens different compounds



Thin film in most

$\gamma = 0$

HgTe strong SOC, undergoes a transition as a function of film thickness d:



"band inversion"

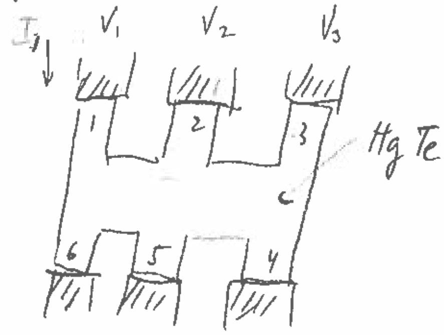
$\Rightarrow \gamma = 1$

2D TI

The experiment measured conductance of the edge states

expect $G = 2\frac{e^2}{h}$ for two ballistic channels.

For a general geometry the currents can be found via the Landauer-Buttiker formula



$$I_i = \frac{e^2}{h} \sum_j \left(T_{ji} V_i - T_{ij} V_j \right)$$

$\sum_i I_i = 0$, T_{ij} - transmission matrix

For ballistic edge states we have

$$T_{i,i+1} = T_{i+1,i} = 1 \,, \quad \text{all others are } 0$$

$$T_{ij} = \left\{ \begin{array}{ll} 1, & i = j \pm 1 \pmod 6 \\ 0, & \text{otherwise} \end{array} \right\} = \# \text{ of edges that connect } i \text{ and } j$$

$(T_{ii} = 0)$

1+1 – one for each edge

Two-terminal measurement $T_{12} = T_{21} = 2$, $T_{11} = T_{22} = 0$

$$I_1 = \frac{e^2}{h} (2V_1 - 2V_2)$$

$$= \frac{2e^2}{h} (\Delta V)$$

$$\boxed{G = \frac{I}{\Delta V} = \frac{2e^2}{h}}$$

Conductance.