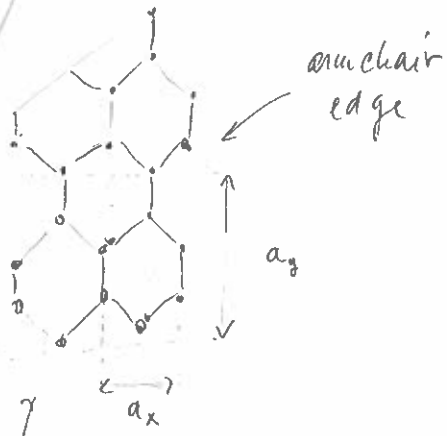
LECTURE 10

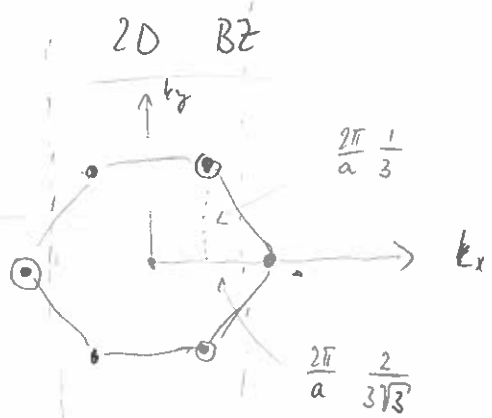
...s of edge states in KH model



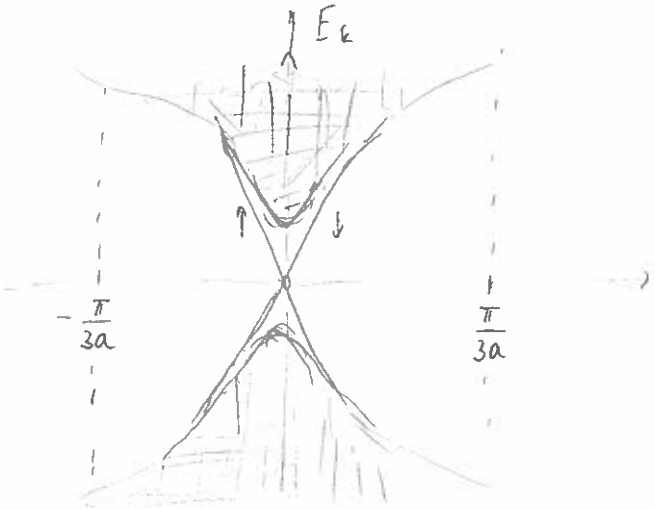
"zigzag" edge

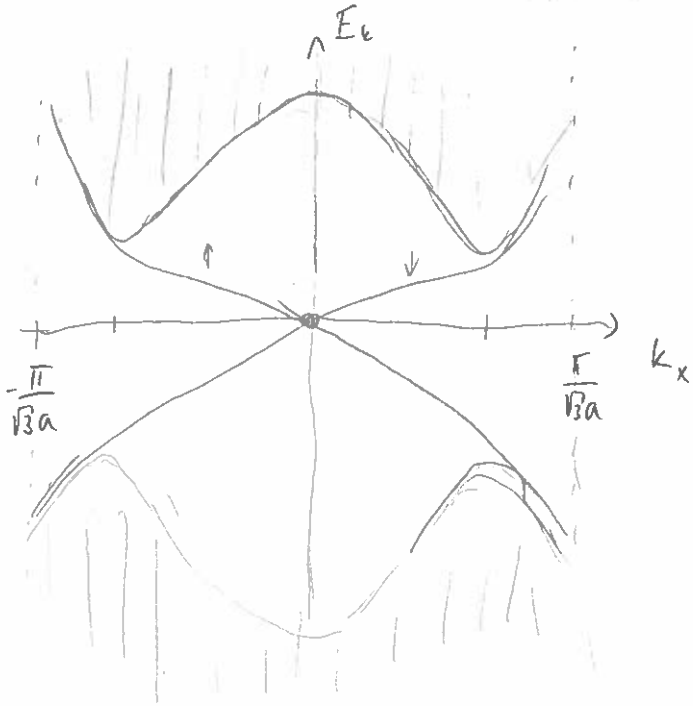
$a_x = \sqrt{3}a$

$a_y = 3a$

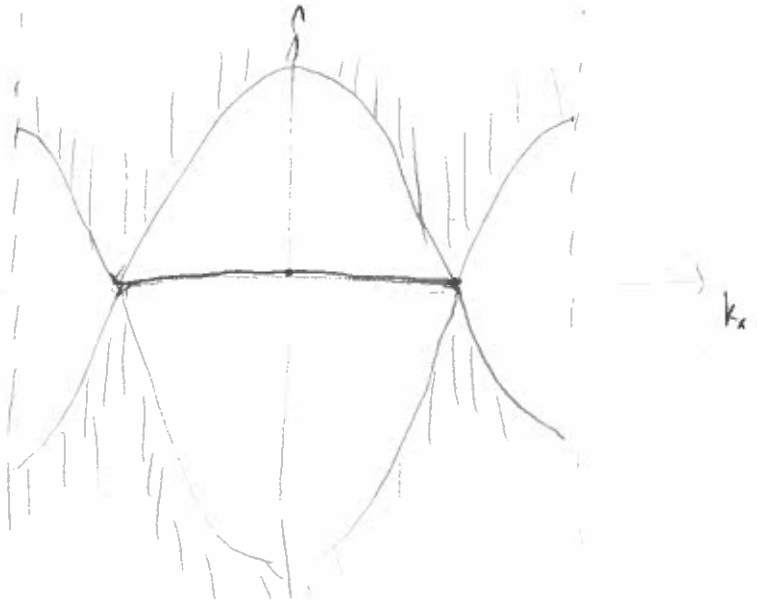


$x = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = a\frac{\sqrt{3}}{2}$





$m_{KH} = 0$



Chern # for spin ↓, ↑

$n = \mathrm{sgn}(v_x v_y m)$

$n_\uparrow = +1 \qquad n_\downarrow = -1$

$n_{tot} = n_\uparrow + n_\downarrow = 0$ ✓ $\mathcal{T}$-invariant system

$\sigma_{xy} = 0$

$n_s = n_\uparrow - n_\downarrow = 2$

Spin Hall conductivity, $\sigma^s_{xy} = \frac{J_s}{E_x}$

J_s - spin current: $\vec{J}_s = \left(\frac{1}{2}\hbar\right)\left(\frac{\vec{J}_\uparrow - \vec{J}_\downarrow}{e}\right)$

$$\sigma^s_{xy} = \frac{\hbar}{2e}\left(\sigma^\uparrow_{xy} - \sigma^\downarrow_{xy}\right) = \frac{\hbar}{2e}\frac{e^2}{h}\left(n_\uparrow - n_\downarrow\right) = \frac{e}{2\pi}$$

Note, however, that since spin is not conserved in general, Quantization of σ^s_{xy} is NOT ROBUST.

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The edge states ARE ROBUST protected by a Z_2 topological invariant that characterizes $\mathcal{T}$-invariant band structure in 2D.

The Z_2 invariant for Quantum Spin Hall states

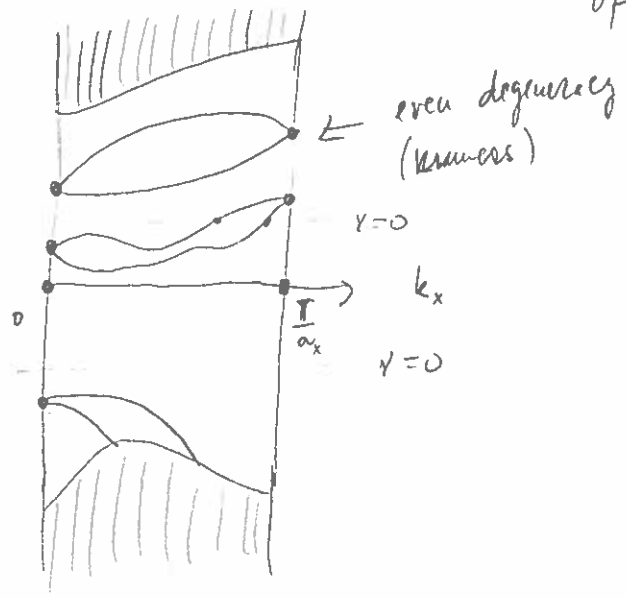
[Kane & Mele, PRL 95, 146802 (2005)]

Consider the edge of a 2D T-invariant insulator.

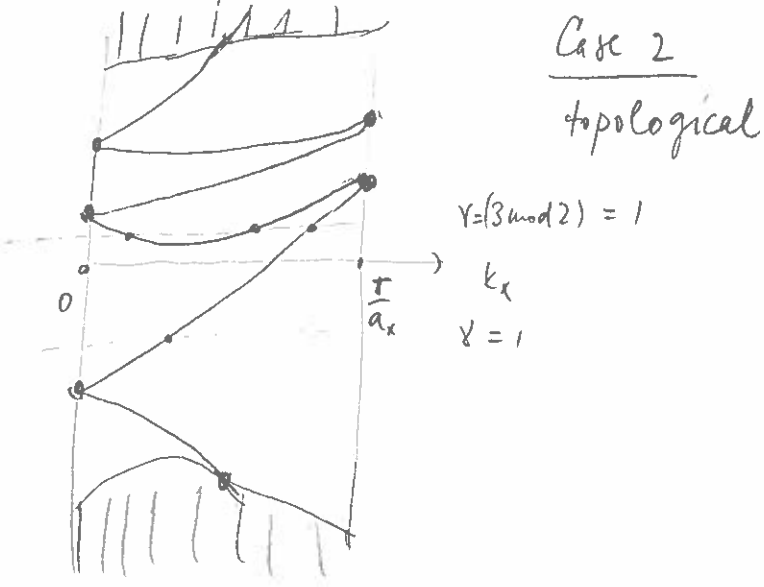
There exist 2 possibilities for edge states.

T: $E_k = E_{-k}$ ⇒ look only at the right half of the surface BZ

Case 1
trivial



Case 2
topological



- here, surface states exist but are not protected.

- surface states are topologically protected
⇒ i.e. cannot be removed without breaking T or closing the bulk gap.

Define an index: ν = (# of Fermi points between 0 and $\frac{\pi}{a_x}$) mod 2

Z_2 index, $\nu = 0, 1$

All 2D T-invariant insulators are characterized by a $\mathbb{Z}_2$-valued index ν, with $\nu=0$ indicating a trivial phase while $\nu=1$ is a topological phase.

- $\nu=1$ implies topologically protected edge states (robust against any T-invariant perturbation)
- ν can change only when bulk T is broken or bulk gap is closed.