P525b **Problem set # 5** **Due: Feb. 28, 12:30pm**

1. (10 points) Quadratic band crossing.

Consider a band crossing described by the 2×2 matrix Hamiltonian

$$\mathcal{H}(\mathbf{q}) = \begin{pmatrix} m & q_x^2 - q_y^2 - 2iq_xq_y \\ q_x^2 - q_y^2 + 2iq_xq_y & -m \end{pmatrix},$$

where m is a constant parameter.

a) Find the spectrum and show that $\mathcal{H}(\mathbf{q})$ describes a quadratic crossing between two bands as m goes through zero.

b) If the Chern numbers of the two bands are zero for $m > 0$ what are they when $m < 0$?

2. (15 points) Mercury Telluride quantum well via parity eigenvalues.

The effective theory of the first experimentally discovered 2D topological insulator HgTe is given by the following Bloch Hamiltonian on the simple square lattice

$$\mathcal{H}(\mathbf{k}) = \lambda\sigma_z(s_x \sin k_y - s_y \sin k_x) + \sigma_x M_{\mathbf{k}}.$$

Here $M_{\mathbf{k}} = \epsilon - 2t(\cos k_x + \cos k_y)$ and $\mathbf{s}$, $\boldsymbol{\sigma}$ represent the Pauli matrices in spin and orbital space, respectively. λ, ϵ and t are model parameters. Hereafter, we take $\lambda = 1$ and measure ϵ and t in units of λ.

a) Find the spectrum of $\mathcal{H}(\mathbf{k})$. Show that $\mathcal{H}(\mathbf{k})$ respects both $\mathcal{T}$ and $\mathcal{P}$. (The inversion operation here involves exchange of the two orbitals on the same site implemented by σ_x).

b) Use the Fu-Kane criterion to classify the topological phases of $\mathcal{H}(\mathbf{k})$ when the two negative-energy bands are filled. It is most instructive to fix t to a positive value and sketch the phase diagram as a function of ϵ. Alternately you can give a phase diagram in the ϵ-t plane. Please label clearly all the phase transitions and assign the Z_2 index to the gapped phases as appropriate.

c) If the inversion symmetry of the physical system is broken (e.g. by the substrate) then the following $\mathcal{T}$-invariant term becomes allowed

$$\delta\mathcal{H} = \sigma_y s_z m,$$

with m a constant parameter. Find the spectrum of the Hamiltonian for $m \neq 0$. Use adiabatic continuity arguments to deduce the phase diagram for small $|m|$. Make a sketch similar to part (b) and pay particular attention to any differences that arise due to non-zero m.