P525b	Problem set # 4	Due: Feb. 7, 12:30pm

1. (10 points) Charge conjugation symmetry $\mathcal{C}$.

Consider a general Bloch Hamiltonian

$$H = \sum_{\mathbf{k}} \mathcal{H}_{\alpha\beta}(\mathbf{k}) c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\beta}$$

where α and β are basis and orbital indices. Charge conjugation (also known as particle-hole) transformation $\mathcal{C}$ is defined as

$$c_{\mathbf{k}\alpha} = U_{\alpha\beta}(\mathbf{k}) d^\dagger_{-\mathbf{k}\beta}.$$

The Hamiltonian is said to be particle-hole symmetric if a transformation $U_{\alpha\beta}(\mathbf{k})$ can be found so that H is the same when expressed in terms of electron operators c and hole operators d.

a) Similar to our discussion of $\mathcal{T}$ and $\mathcal{P}$ in class find the condition that must be imposed on $\mathcal{H}(\mathbf{k})$ so that H is p-h symmetric.

b) Show that, for a system with p-h symmetry defined as above, the energy eigenvalues come in pairs $(E_{\mathbf{k}}, -E_{\mathbf{k}})$, when in addition either $\mathcal{T}$ or $\mathcal{P}$ is present.

c) Find the representation of $\mathcal{C}$ for the spinless graphene Hamiltonian discussed in class.

2. (10 points) Domain wall in the Semenoff mass.

Consider the low-energy theory of spinless graphene as derived in class with the Semenoff mass m_S. Analyze the structure of the low-energy electron states associated with a domain wall in $m_S(x)$, such that $m_S(x) \to \pm m_0$ as $x \to \pm\infty$. How many gapless modes are there? Sketch the energy spectrum as a function of k_y.