QUANTUM FIELD THEORY II: Topics for the COURSE
(Phys 508) Jan-April 2015

It is assumed that those taking this course will have taken an introductory course in field theory and/or advanced QM, and are thereby familiar with path integrals, second quantization, elementary free field theory, and some Quantum Electrodynamics. The best possible preparation would be PHYS 526, henceforth known as QFT I, which was taught in autumn 2014 by G Semenoff. **I will cover SOME of the topics listed below (there will certainly NOT be time to do all of them!).** Part A covers the methods and part B the applications. How much of Part A we will cover will depend on the background of the students.

(A) QUANTUM FIELD THEORY: The partition function/generating functional Z[j], as a functional of external source j(x). Expansion of Z[j] in cumulants/connected graphs, and in correlation functions; n-point propagator. Perturbation expansion – derivation of diagrammatic rules and diagrams from Z[j]. Schwinger-Dyson equation, and the S-matrix; Keldysh contours. Schwinger-Dyson in diagrams. Semiclassical expansion and connection to tree graphs, loop expansions; instanton methods.

Background field method; integrating out auxiliary fields. Decoupling slow and fast variables – gradient expansions and eikonal expansions; effective potentials. Important examples: scalar ϕ^4 theory, coupled complex fields. Path integrals for fermionic fields. Path integrals in the presence of constraints. Path integrals for fermionic fields, with as examples, Dirac electrons & electron gas. Path integrals for gauge fields: constraints and the Fadeev-Popov determinant, applied to QED, Yang-Mills theory, and gravity.

Dynamics of density matrix – Feynman-Vernon theory, influence functionals. Spectral functions for propagators, analytic properties in complex plane, dispersion relations. Scattering functions and cross-sections; unitarity. Application to polarons, superconductors, magnets, QED, and quantum cosmology

Renormalization: power-counting and renormalizability, counterterms, loop expansions, & dimensional regularization. Ward identities. Calculating graphs, doing renormalization for concrete examples: Kondo problem, ϕ^4 theory, QED, electron gas, Yang-Mills, gravity.

(B) APPLICATIONS in CONDENSED MATTER & HIGH-ENERGY THEORY: The course will use examples drawn from both condensed matter physics and high-energy physics. The following is an over-complete list (there will certainly not be enough time to cover all of these):

Quantum Diffusing Particle: influence functional, Z[j], dynamics of density matrix, for a particle coupled to a quantum environment.

Interacting Bosonic Fields: Non-interacting phonons, effect of interactions on propagator. Interacting hard-core bosons – form of Z[j] and correlation functions, spontaneous symmetry-breaking instability to superfluid phase, and propagators in superfluid phase. Charged superfluid bosons and the Anderson/Brout-Englert-Higgs mode.

Interacting Fermionic Fields: The interacting electron gas, and QED – the parallels. Fermi liquid theory, quasiparticles, 4-point vertices and the Landau-Boltzmann equation. 2-d graphene- effective field theory. The instability to superfluidity – superfluid He-3 and superconductors. The effect of disorder – localization phenomena.

Gauge Theories: QED as a simple gauge theory; form of Z[j], and of 3- and 4-point vertices; Ward identities. Infra-red divergences, treated perturbatively and using background field methods; eikonal expansion. Scattering amplitudes, cross-sections. High-energy scattering and eikonal expansion. Non-Abelian gauge theories: superfluid He-3, and the standard model - parallels. Form of Z[j], the Fadeev-Popov and BSRT quantization, ghosts, for Yang-Mills theory. Slavnov-Taylor identities. Feynman rules, dimensional regularization of graphs. Quantum gravity: spin-2 interacting gravitons, the connection to geometric ideas. Non-renormalizability of gravity.

Spin problems: Path integral for spin, gauge symmetries, lattice gauge theories. Phenomenology and effective field theories of ferromagnets and antiferromagnets. Magnetic solitons, Sine-Gordon model, domain walls, topological field theories for integer and half-integer spins.