Phys 503+508: HOMEWORK ASSIGNMENT No (3)

Tuesday March 25th 2014

DUE DATE: Thursday April 10th 2014.

Assignments handed in late will not receive a full mark.

QUESTION (1): EQUATIONS OF MOTION

(i) Consider a 'toy model' for a bosonic scalar field, in which we collapse the fields to points. The diagram rules for this theory assign a factor $i\hbar$ to a free line (ie., $\Delta_F(x, x') \rightarrow 1$), and a factor $-ig_o/\hbar$ to the bare 4-point vertex. Now, working back from these rules, find the action for this theory, and an expression for the generating function $\mathcal{Z}[J]$ of this in terms of a set of n-point correlators. What is the Schwinger-Dyson equation of motion for $\mathcal{Z}[J]$ in this theory, and what is its solution when $g_o = 0$?

Finally, construct the Schwinger-Dyson hierarchy of equations of motion for this theory, by successive differention of your equation of motion for $\mathcal{Z}[J]$.

(ii) Consider a simple non-relativistic fermion theory in which the interaction between the fermions is a constant 'point interaction' V_o in energy-momentum space. Starting from the full Bethe-Salpeter equation for the 4-point vertex $\Gamma_4(\mathbf{k}, \mathbf{k}', \mathbf{q}; \epsilon, \epsilon', \omega)$ for a non-relativistic fermion system. derive a simple equation for $\Gamma_4(\mathbf{k}, \mathbf{k}', \mathbf{q}; \epsilon, \epsilon', \omega)$ in this simple 'point-interaction' model. Then use this result to derive the standard 'RPA' result for the 1-particle self-energy of the fermions in this theory.

QUESTION (2): CALCULATING DIAGRAMS

A full calculation of some diagram, including integration over all free variables, is very lengthy for all but simple diagrams. But we can illustrate important ideas by just doing the frequency integrations for non-relativistic diagrams - we do this here for several diagrams involving non-relativistic fermions.

(i) Show that the finite-T diagram in Fig. 1, a contribution to the thermodynamic potential of the system, can be written as

$$\Delta\Phi = -V_o^3 \sum_{\mathbf{k_1},\mathbf{k_2},\mathbf{k_3},\mathbf{q}} \frac{1}{2}\,\mathbf{P}\,\frac{f_1(1-\bar{f}_1)(f_2-\bar{f}_2)(f_3-\bar{f}_3)}{(\Omega_1-\Omega_2)(\Omega_1-\Omega_3)} + \frac{\pi^2}{6}\, f_1(1-\bar{f}_1)(f_2-\bar{f}_2)(f_3-\bar{f}_3)\,\delta(\Omega_1-\Omega_2)\delta(\Omega_1-\Omega_3) \qquad (1)$$

where we have assume that the 4-fermion interaction here (shown as a red hatched line) is a constant V_o in energy-momentum space. The notation here is: $f_j = f(\epsilon_{\mathbf{k_j}})$ and $\bar{f}_j = 1 - f(\epsilon_{\mathbf{k_j-q}})$, where $f(\epsilon_{\mathbf{k}})$ is the Fermi function; $\Omega_j = (\epsilon_{\mathbf{k_j}} - \epsilon_{\mathbf{k_j-q}})$; and $\mathbf{P}$ denotes the principal value.

(ii) Now using the Landau-Cutkowsky rules, find expressions for the self-energy graphs $\Sigma(\mathbf{p}, \epsilon)$ graphs in Fig. 2 and 3, in which you have integrated over internal frequencies. In Fig. 2, assume that the bare interaction is a constant V_o; and in Fig. 3, assume that the effective interaction $I_6(\mathbf{k}, \mathbf{k}', \mathbf{q}; \epsilon, \epsilon', \omega)$ reduces to a constant I_o.

FIGURES ON NEXT PAGE

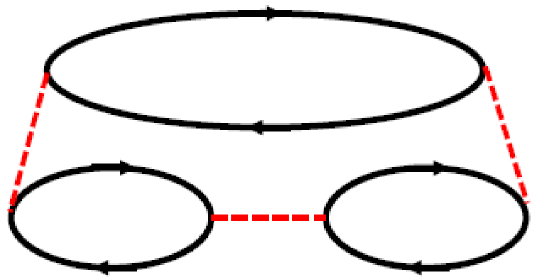

Fig. 1

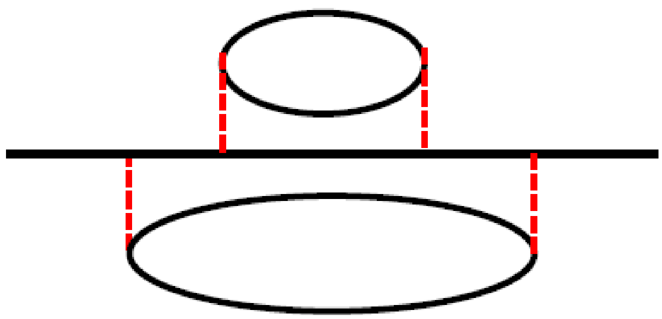

Fig. 2

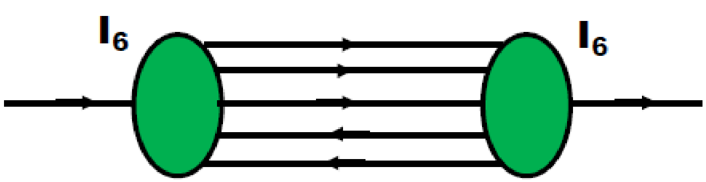

Fig. 3

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