Phys 503+508: HOMEWORK ASSIGNMENT No (2)

Tuesday March 4th 2014

DUE DATE: Wednesday March 19th 2014.
Assignments handed in late will not receive a full mark.

QUESTION (1): DIAGRAMS for $\phi^3(x)$ THEORY

You are familiar from the notes with $\phi^4(x)$ theory. Consider now a scalar field theory where the Lagrangian density is

$$L(\Phi) \;=\; \frac{1}{2}[\partial_\mu \phi(x) \partial^\mu \phi(x) - m^2 \phi^2(x)] \;-\; \frac{g_o}{3!}\phi^3(x) \tag{1}$$

(i) Write down the Feynman rules for this theory. Then give expressions for the generating functional $\mathcal{Z}[J]$, the generating functional $W[J]$, and the connected correlator $\mathcal{G}_2(x,x')$ for this theory.

(ii) Calculate, starting from the expression for $\mathcal{G}_2(x,x')$ you have given above, the terms in $\mathcal{G}_2(x,x')$ up to order g_o^2. Draw the diagrams corresponding to each term in this, and show how they correspond to the Feynman rules you have given.

QUESTION (2): COUPLED SCALAR FIELDS

This problem will help you to understand functionals better, and the expansions they generate.

(i) Define the Gaussian functional $F[\phi]$ and the quadratic shift operator $\hat{Q}_2$ by

$$\begin{aligned} F[\phi] \;&=\; e^{\frac{i}{\hbar}\int dx_1 \int dx_2 \; \frac{1}{2}[\phi(x_1)\Delta_F(x_1,x_2)\phi(x_2)] \,+\, \int dx J(x)\phi(x)} \\ \hat{Q}_2[\phi] \;&=\; e^{\frac{i}{\hbar}\int dx \int dx' \; \frac{1}{2}\frac{\delta}{\delta\phi(x)} K(x,x') \frac{\delta}{\delta\phi(x')}} \end{aligned} \tag{2}$$

and now derive an expression for $\hat{Q}_2 F[\phi]$ (see eqtns. (76)-(78) of the notes on functionals).

(ii) Now let's consider a pair of free scalar fields $\chi(x)$ and $\phi(x)$, with masses M_o and m_o respectively, coupled by a term $-\lambda_o \phi^2(x)\chi(x)$ in the Lagrangian density. If $\phi(x)$ is coupled to an external current $J(x)$, we know that in the semi-classical or 'loop-free' limit, the generating functional $\mathcal{Z}[J]$ for the ϕ-field after the χ-field has been integrated out is

$$\mathcal{Z}[J] \;=\; e^{\frac{i}{\hbar}\int dx \int dx' \; \frac{1}{2}\frac{\delta}{\delta\psi(x)} \Delta_F^{\chi}(x,x') \frac{\delta}{\delta\psi(x')}} \; \tilde{\mathcal{Z}}^o_\psi[J] \,|_{\psi=0} \tag{3}$$

where $\tilde{\mathcal{Z}}^o_\psi[J]$ takes the form of the free field generating functional $\mathcal{Z}_o[J]$ except now, in place of the usual free field propagator $\Delta_F(x,x')$, we have a propagator $\tilde{\Delta}(x,x'|\psi)$ satisfying the differential equation

$$(\partial^2 + m_o^2 + \frac{1}{2}\lambda_o \psi(x)) \; \tilde{\Delta}(x,x'|\psi) \;=\; -\delta(x-x') \tag{4}$$

(compare eqtns. (127) and (130) in the notes)).

Now derive an expression for the connected 4-point propagator $\mathcal{G}_4(x_1,x_2,x_3,x_4)$ for this system, and derive the lowest-order term (of order λ_o^2) for this connected 4-point function. Show this graphically; and then show graphs for all the terms that contribute to $\mathcal{G}_4(x_1,x_2,x_3,x_4)$ up to order λ_o^4 (no need to derive them however).

END of QUESTION SHEET 2