Phys 503+508: HOMEWORK ASSIGNMENT No (1)

Thursday January 23rd 2014

DUE DATE: Wednesday Feb 5th 2014.
Assignments handed in late will not receive a full mark.

QUESTION (1): SIMPLE HARMONIC OSCILLATOR

Suppose we have a simple 1-d harmonic oscillator with Lagrangian

$$\mathcal{L} \;=\; \frac{m}{2}(\dot{x}^2 - \Omega_o^2 x^2) \tag{1}$$

where $x(t)$ is the oscillator coordinate.

(i) Find, using any method you like, the propagator $G_o(\mathbf{r}, \mathbf{r}'; t, t')$, in real spacetime, for this non-relativistic free oscillator. For the prefactor, if you use the path integral method, you can use the identity

$$z/\sin z \;=\; \prod_{n=1}^{\infty} [1 - z^2/\pi^2 n^2]^{-1} \tag{2}$$

(ii) Now suppose the initial wave-function, as $t = 0$, is the Gaussian wave-packet

$$\psi(x, t = 0) \;=\; \frac{1}{(2\pi L)^{1/4}} \exp \frac{i}{\hbar} \left[mux - \frac{\hbar x^2}{4iL} \right] \tag{3}$$

Find the solution $\psi(x, t)$ for all times.

QUESTION (2): DRIVEN OSCILLATOR

Suppose now we have a simple 1-d harmonic oscillator driven by some arbitrary time-dependent force $J(t)$, with Lagrangian

$$\mathcal{L} \;=\; \frac{m}{2}(\dot{x}^2 - \Omega_o^2 x^2) - J(t)x \tag{4}$$

where again $x(t)$ is the oscillator coordinate.

(i) Find the propagator $G_o(\mathbf{r}, \mathbf{r}'; t, t'|J(t))$, a functional of the external $J(t)$. You can assume that the prefactor is the same as for the undriven oscillator - so you only have to find the classical action. The answer and hints are given in the notes - you have to derive the answer!

(ii) Find the second functional derivative of this expression with respect to $J(t)$; and then, taking the limit as $J \to 0$, find the correlator $G_2^o(x, x'; t, t')$.

(iii) Then, show that the "vacuum amplitude' $G_{oo}(t|J) = \langle 0|\hat{G}(t)|0\rangle_J$ from the ground state $|0\rangle$ of the free oscillator at time 0, to the same state at time t, is given in the presence of the force $J(t)$ by

$$G_{oo}(t|J) \;=\; \exp\left[-\frac{i}{2m\hbar\Omega_o} \int_0^t d\tau \int_0^{t'} d\tau' J(\tau) J(\tau') e^{i\Omega_o(\tau - \tau')} \right] \tag{5}$$

END of QUESTION SHEET 1