QUANTUM FIELD THEORY in CONDENSED MATTER & HIGH-ENERGY PHYSICS
(Combination of Phys 503 & Phys 508)

PATH INTEGRAL THEORY of QUANTUM MECHANICS: The basic idea of path integrals and of the 1-particle propagator/Green function. Formulation in terms of discretized paths. Semiclassical expansion: calculations of prefactor, & of classical exponent. The effect of constraints on path integrals. Model examples: free particle, simple harmonic oscillator, particle in a magnetic field – on a ring and in 2-d and 3d space; particle in a monopole field. Propagator for a spin-1/2 system. Analytic properties of propagator in the complex time plane; spectral representation.

Perturbation expansions – Feynman diagrams, Lippmann-Schwinger eqtn, & Dyson equation.

Propagator for particle in an external dynamic 'noise' or 'source' field: exact solution, perturbation expansion in interaction, Born-Oppenheimer approx. Berry's phase in adiabatic limit.

QUANTUM FIELD THEORY: The relationship between 1-particle path integral and quantum field theory for the partition function/generating functional Z[j], as a functional of external source j(x). Expansion of Z[j] in cumulants/connected graphs, and in correlation functions; n-point propagator. Effect of interactions on the generating functional. Schwinger-Dyson equation, and the S-matrix; Keldysh contours. Background field method; integrating out auxiliary fields. Decoupling slow and fast variables. Legendre transformation to effective potential. Important model examples: scalar ϕ^4 theory, coupled complex fields. Path integrals for fermionic fields. Path integrals in the presence of constraints. Path integrals for fermionic fields, with as examples, Dirac electrons & electron gas. Path integrals for gauge fields: constraints and the Fadeev-Popov determinant.

Perturbation expansion – derivation of diagrammatic rules and diagrams from Z[j]. Schwinger-Dyson in diagrams. Semiclassical expansion and connection to tree graphs, loop expansions; instanton methods. Some other non-perturbative expansions (1/N, eikonal, canonical transformation). Dynamics of density matrix – Feynman-Vernon theory. Spectral functions for propagators, analytic properties in complex plane, dispersion relations. Scattering functions and cross-sections; unitarity. Renormalization: power-counting and renormalizability, counterterms, loop expansions, & dimensional regularization. Ward identities.

Model examples: scalar field, QED, disordered fermions, interacting electron gas, quantum diffusing particle. Some other important Lagrangians.

APPLICATIONS in CONDENSED MATTER & HIGH-ENERGY THEORY: The role of symmetries in determining the Lagrangians - why CM physics, stat mech, and high-energy physics are so closely related.

Quantum Diffusing Particle: influence functional, Z[j], dynamics of density matrix.

Interacting Bosonic Fields: The simple case of interacting phonons. Z[j] for this interacting bosonic field: form of correlators, relationship to scalar ϕ^4 theory. Propagator for non-interacting phonons, effect of interactions on propagator; structural instability. Probing phonons with neutrons. Interacting hard-core bosons – form of Z[j] and correlation functions, spontaneous symmetry-breaking instability to superfluid phase, and propagators in superfluid phase. Charged superfluid bosons and the Anderson/Brout-Englert-Higgs mode. The effect of disorder and defects on propagating bosons.

Interacting Fermionic Fields: The interacting electron gas, and QED – the parallels. Interacting fermions in 3 dimensions – Fermi liquid theory, quasiparticles, 4-point vertices and the Landau-Boltzmann equation. 2-d graphene- effective field theory. Response functions and experiments. The instability to superfluidity – superfluid He-3 and superconductors. The effect of disorder – localization phenomena. QED as a simple gauge theory; form of Z[j], and of 3- and 4-point vertices; Ward identities. Infra-red divergences, treated perturbatively and using background field methods; eikonal expansion. Scattering amplitudes, cross-sections. High-energy scattering and eikonal expansion.

Non-Abelian Gauge Theory: Superfluid He-3, and the standard model - parallels. Form of Z[j], the Fadeev-Popov and BSRT quantization, ghosts, for Yang-Mills theory. Slavnov-Taylor identities. Basic elements of the standard model – Higgs boson, anomalies, Feynman rules, dimensional regularization of graphs. Quantum gravity: spin-2 interacting gravitons, non-renormalizability. Superfluid He-3: form of Z[j], and response functions. Collective modes, identification with equivalent modes in standard model + gravitons.