D.2. SPIN DYNAMICS

In what follows we will eschew much of the standard material on spin dynamics, to be found in elementary texts. In fact much of this material is adapted to rather specific & in most cases fairly elementary problems. Once one starts to consider more general problems, it rapidly becomes clear that many of these are very hard to solve, and even their qualitative features can be very subtle.

Amazingly, many of these features appear already in the simple spin-½ system. We therefore begin with this, and find out just how complex it can be. It turns out to be a very useful model for a deeper study of the time-dependent Schrodinger equation, and brings out very clearly many of the topological features of this. We look only at a single spin-½ here.

The problem of a higher spin S is of course much more complex. After a cursory look at the different kinds of problem one can meet, we focus on the phenomenon of spin tunneling - this is of pedagogical interest because it allows us to compare the use of perturbation methods, WKB, path integral (instanton, and exact diagonalization methods, thereby clarifying the relationship between all of these.

Many of the results of the model problems discussed here are "inputs" into more complex models involving many particles. We will note two of these; the application to models of decoherence, and the study of quantum chaos.

Finally, note we are not concerned in this section with problems of 2 or more spins (which involve us in essential questions about entanglement). This comes in section D.3.

D.2.1. SPIN - ½ DYNAMICS

As we already noted above, the most general Hamiltonian for a spin-½ is extremely simple. We assume a general time-dependent field $\underline{B}(t)$, and a Hamiltonian

$$\mathcal{H}(t) = -\frac{\gamma}{2}\underline{B}(t)\cdot\hat{\underline{\tau}} \equiv \hbar\underline{B}(t)\cdot\hat{\underline{\tau}} \qquad (86)$$

Then the Schrödinger eqtn. takes the form

$$\mathcal{H}(t)\,\chi_\sigma(t) = i\hbar\,\partial_t\,\chi_\sigma(t) \qquad (87)$$

where $|\chi_\sigma(t)\rangle = |\sigma\rangle$ are spinor wave-functions; writing these explicitly,

$$\left.\begin{aligned} i\dot{\chi}_+(t) &= B_z(t)\chi_+(t) + B_-(t)\chi_-(t) \\ i\dot{\chi}_-(t) &= -B_z(t)\chi_-(t) + B_+(t)\chi_+(t) \end{aligned}\right\} \qquad (88)$$

where we define : $$B_\pm(t) = B_x(t) \pm iB_y(t) \quad (89)$$

We thus have 2 coupled 1st-order differential eqtns, which reduce to the 2nd-order eqtn:

$$B_-(t)\ddot{\chi}_+(t) - \dot{B}_-(t)\dot{\chi}_+(t) + \left[B_-(t)\left(|\underline{B}(t)|^2 + i\dot{B}_z(t)\right) - i\dot{B}_-(t)B_z(t)\right]\chi_+(t) = 0 \quad (90)$$

and a corresponding equation for $\chi_-(t)$ (which is of course not independent, since $|\chi_+|^2 + |\chi_-|^2 = 1$), in which $B_-(t) \to B_+(t)$, and $B_z(t) \to -B_z(t)$.

We note that this equation (90) is in general completely unsolvable - it is a 2nd order eqtn in which the coefficients depend on t. It can also have extremely complicated solutions, even when the behaviour of $\underline{B}(t)$ is fairly simple (including, eg., quantum chaotic behaviour).

In what follows we will first look at some of the general features of this problem, and then study a number of examples of particular interest.

D.2.1.(a) GENERAL RESULTS for SPIN DYNAMICS

One may discern interesting general features of the problem by looking at both the Schrodinger eqtn and the path integral formulation of the problem; and the adiabatic and sudden limits are of particular general interest. We consider things from all these points of view

(1) SCHRODINGER EQTN : LIMITING BEHAVIOUR : From eqtn. (90) above, we see that there are several important limiting cases where we expect the dynamics to yield to analytic methods. We classify these as follows:

(a) Longitudinal limit : Suppose $b_\pm(t)$ are always small (with no condition necessarily implied for $\dot{B}_\pm(t)$, which may vary fast even though it is small). Then we write
$$B_\pm(t) \to \epsilon\mathcal{B}_\pm(t) \quad (91)$$

and the differential eqtn becomes

$$\epsilon\mathcal{B}_-(t)\ddot{\chi}_+(t) - \dot{\mathcal{B}}_-(t)\dot{\chi}_+(t) + \left[\epsilon\mathcal{B}_-(t)\left(|\underline{B}(t)|^2 + i\dot{B}_z(t)\right) - i\dot{\mathcal{B}}_-(t)B_z(t)\right]\chi_+(t) = 0 \quad (92)$$

This equation can be solved by perturbative techniques. Depending on the form of the functions $\mathcal{B}_-(t)$ and $B_z(t)$, we can use either boundary value analysis or a WKB analysis (unless there are secular terms, in which case one can attempt a multiple-scale analysis). Full details of this are beyond the scope of these notes, but they are discussed in an Appendix.

(b) Adiabatic limit: Suppose that the time derivative $\dot{\underline{b}}(t)$ is small, so we have that

$$\left.\begin{aligned} \dot{B}_-(t) &\to \epsilon\,\dot{B}_-(t) \\ \dot{B}_z(t) &\to \epsilon\,\dot{B}_z(t) \end{aligned}\right\} \qquad (93)$$

We then write the Schrodinger eqtn. as

$$B_-(t)\,\ddot{\chi}_+(t) - \epsilon\dot{B}_-(t)\dot{\chi}_+(t) + \left[B_-(t)\left(|\underline{B}(t)|^2 + i\epsilon\dot{B}_z(t)\right) - i\epsilon B_z(t)\,\dot{B}_-(t)\right]\chi_+(t) = 0 \qquad (94)$$

In this equation we have essentially a harmonic oscillator with time-dependent frequency/mass, and a very weak frictional force. Actually this problem is not at all trivial, and indeed the adiabatic limit is non-trivial for a spin ½ system.

Note that if both $\dot{\underline{B}}(t)$ and $B_-(t)$ are small (ie., the field not only fluctuates slowly, but stays close to $\hat{z}$), then the problem simplifies still further. Suppose we write

$$\left.\begin{aligned} B_-(t) &\to \epsilon_1 B_-(t) \\ \dot{\underline{B}}(t) &\to \epsilon_2\,\dot{\underline{B}}(t) \end{aligned}\right\} \qquad (95)$$

where $\epsilon_1, \epsilon_2 \ll 1$, but are independent of each other. Then we have

$$\epsilon_1 B_-(t)\ddot{\chi}(t) - \epsilon_2\dot{B}_-(t)\dot{\chi}_+(t) + \left[\epsilon_1|\underline{B}(t)|^2 B_-(t) - i\epsilon_2 B_z(t)\,\dot{B}_-(t) + i\epsilon_1\epsilon_2 B_-(t)\dot{B}_z(t)\right]\chi_+(t) = 0 \qquad (96)$$

and then we can let $\epsilon_1, \epsilon_2 \to 0$ in either order - notice that the term $\sim O(\epsilon_1\epsilon_2)$ can be neglected in either of these 2 cases.

(c) Fast Fluctuating limit: Suppose now we know that the field is changing very fast, so that passage through nearly degenerate levels have little effect. To implement perturbation approximations here, we write

$$\dot{\underline{B}}(t) \to \dot{\underline{B}}(t)/\epsilon \qquad (97)$$

so that the differential eqtn now becomes

$$\epsilon B_-(t)\,\ddot{\chi}_+(t) - \dot{B}_-(t)\dot{\chi}_+(t) + \left[\epsilon|\underline{B}^2(t)|\,B_-(t) + i\left(B_-(t)\dot{B}_z(t) - B_z(t)\dot{B}_-(t)\right)\right]\chi_+(t) = 0 \qquad (98)$$

which again can be handled using boundary value, WKB, or multiple scale analysis.

We notice 2 obvious defects in the simple classification given here. The first is that the form of the O.D.E. in (90), and the subsequent classification, is very much tied to the assumption of a quantization along the z-axis in (87)-(90). This starting assumption only makes sense if the Hamiltonian itself in (86) singles out the $\hat{z}$-axis for special consideration (the most obvious case being if $b_z \gg b_-$ at all times, which is the case of the longitudinal limit described by (91) and (92); or if the asymptotic behaviour of $\underline{b}(t)$ when $|t| \to \infty$ is along $\hat{z}$).

The 2nd defect is obvious from the first. In many cases we will be able to separate $\underline{B}(t)$ into 2 components, of form

$$\underline{B}(t) = \underline{B}_0(t) + \underline{b}(t) \qquad (99)$$

where either $|\underline{B}_0| \gg |\underline{b}|$, or $|\underline{b}(t)|$ fluctuates rapidly compared to $\underline{B}_0(t)$, or both.

In either case it makes sense to solve the problem first in a frame which is co-moving with $\underline{B}_0(t)$, and then add in the field $\underline{b}(t)$ as a correction to it.

This casts the spotlight on the adiabatic limit, which we look at now, in the broader context of the solution of problems where we follow the field in the co-rotating frame.

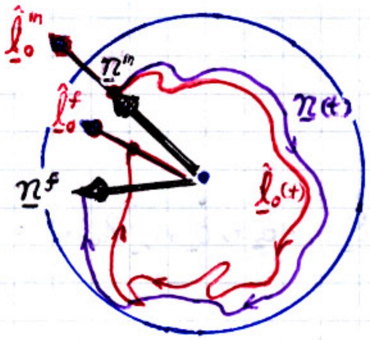
(ii) SLOW DYNAMICS for a SPIN-½ :

We consider the dynamics of a spin-½ in the adiabatic regime, where $\underline{B}(t)$ is written in the form (99), and we assume that $\underline{B}_0(t)$ is varying slowly (the precise condition is fixed below). We neglect any correction $\underline{b}(t)$ which is not slowly varying.

Both the magnitude & direction of $\underline{B}_0(t)$ vary - we write

$$\underline{B}_0(t) = B_0(t)\,\hat{\underline{l}}_0(t) \qquad (100)$$

where $|\hat{\underline{l}}_0(t)| = 1$. We parametrize $\hat{\underline{l}}_0(t)$ by angles $\Theta_0(t), \Phi_0(t)$. In the adiabatic limit we expect the spin $\underline{\tau}$ to follow the same path as $\hat{\underline{l}}_0(t)$, but at any finite rate of change, there will be weak departures. We write, in a way consistent with the preceding section (see, e.g., eqtn. (31)), that



$$\langle \underline{n}(t) | \hat{\underline{\tau}} | \underline{n}(t) \rangle = \underline{n}(t) \qquad (101)$$

for a coherent state $|\underline{n}(t)\rangle$ describing the spin; and we parametrize the vector $\underline{n}$ by angles $\theta(t), \varphi(t)$, as in section D.1.1 (see eqtn (47)).

The situation is then as shown at left; the spin tries to follow the field, but if $\underline{d}_0(t)$ changes too quickly, it cannot follow exactly. Note that for a spin the path $\underline{n}(t)$ may be very complex; we will see this presently.

Defining the Cartesian components of $\underline{B}_0(t)$ as $(B_0^x, B_0^y, B_0^z) = B_0(t)\,(x_0(t), y_0(t), z_0(t))$, we write the Hamiltonian as

$$\left.\begin{aligned} \mathcal{H}_0(t) &= \hbar \underline{B}_0(t)\cdot\hat{\underline{\tau}} = \hbar B_0(t)\begin{pmatrix} z_0(t) & x_0(t)-iy_0(t) \\ x_0(t)+iy_0(t) & -z_0(t)\end{pmatrix} \\ &\equiv \hbar B_0(t)\begin{pmatrix} \cos\Theta_0(t) & \sin\Theta_0(t)\, e^{-i\Phi_0(t)} \\ \sin\Theta_0(t)\, e^{i\Phi_0(t)} & -\cos\Theta_0(t)\end{pmatrix}\end{aligned}\right\} \quad (102)$$

Now define the adiabatic eigenstates of this Hamiltonian, $|\underline{u}_\pm(t)\rangle$, with spin projection $\pm\hbar/2$ along the instantaneous axis $\hat{d}_0(t)$ of the field $\underline{B}_0(t)$, and with eigenvalues $E^0_\pm(t)$

$$\mathcal{H}_0(t)\,|\underline{u}_\pm(t)\rangle = E^0_\pm(t)\,|u_\pm(t)\rangle = \pm\hbar B_0\,|u_\pm(t)\rangle \qquad (103)$$

Now, in the adiabatic limit, we expect the states of the system to evolve along with $|\underline{u}_\pm(t)\rangle$, along with an extra Berry phase. We easily verify this; letting the initial state of the spin be

$$|\psi_{in}(t=0)\rangle = |u_+(t=0)\rangle = |\underline{n}^{in}\rangle \qquad (104)$$

Now if we ignore the exponentially small amplitude for the spin to make a transition in the slow regime, we have at time t that the system is in the state

$$\left.\begin{aligned} |\underline{n}_0(t)\rangle &\xrightarrow[\text{ADIABATIC}]{} e^{-i/\hbar\int_0^t dt' E_+(t')}\, e^{i\phi_B^+(t)}\,|\underline{u}_+(t)\rangle \\ &\equiv e^{-i/\hbar\int dt' E_+(t')}\, e^{i\phi_B(t)}\begin{pmatrix}\cos\frac{1}{2}\Theta_0(t)\, e^{-i/2\,\Phi_0(t)} \\ \sin\frac{1}{2}\Theta_0(t)\, e^{i/2\,\Phi_0(t)}\end{pmatrix}\end{aligned}\right\} \quad (105)$$

where the Berry phase term is

$$\left.\begin{aligned} \phi_B^+(t) &= \int_0^t dt'\,\dot{\Phi}_0(t')\cos\Theta_0(t') \\ &= \int_0^t dt'\,\frac{z_0(t)\left(x_0(t)\dot{y}_0(t) - y_0(t)\dot{x}_0(t)\right)}{x_0^2(t)+y_0^2(t)}\end{aligned}\right\} \quad (106)$$

which, if the path for $\underline{B}_0(t)$ is closed, just becomes the solid angle we enclosed by the closed circuit. All of this is familiar from our previous discussion of Berry phases (including the special choice of the $\hat{z}$-axis in the formulae).

To proceed further it is useful to define an adiabatic expansion parameter ε. We "stretch time", by defining a new time $\bar{t} = \varepsilon t$, and consider the Schrodinger eqtn.

$$\underline{B}_0(\varepsilon t)\cdot\hat{\underline{\tau}}\,|\psi\rangle = i\partial_t\,|\psi\rangle \qquad (107)$$

so that the adiabatic evolution of the system is now written as (cf. (105)):

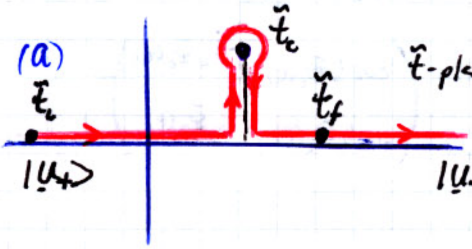
$$\left.\begin{aligned} |\psi(\tilde{t})\rangle \xrightarrow[\varepsilon \ll 1]{} \; e^{-\frac{i}{\hbar\varepsilon}\int_0^{\tilde{t}} d\tilde{t}'\, E_+(\tilde{t}')}\, e^{i\phi_B^+(\tilde{t})}\, |\underline{u}_+(\tilde{t})\rangle \\ \text{where } \tilde{t} = \varepsilon t \end{aligned}\right\} \quad (108)$$

As discussed already for the problem of a general quantum system in chapter C, one can study the properties of this expression, and its asymptotics for small ε; and for a spin-½ this is quite interesting. However it is also rather technical, so it is relegated to an Appendix.

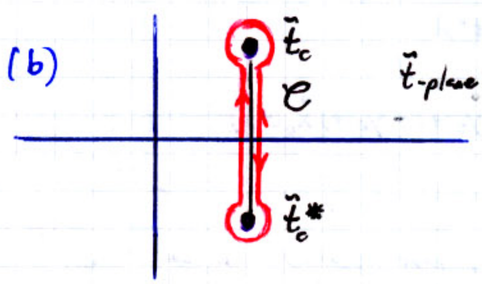
On the other hand we can look very quickly at the exponentially small transition amplitude from $|\underline{u}_+(t)\rangle$ to $|\underline{u}_-(t)\rangle$ in this adiabatic limit, for small ε, using the standard Landau-Dykhne formula. To do this we assume that when $\tilde{t} = \tilde{t}_c$, we have

$$\left.\begin{aligned} \mathcal{H}_o(\tilde{t}_c) &= \hbar \underline{B}_o(\tilde{t}_c)\cdot\hat{\underline{\tau}} = 0 \\ \Delta E_\pm^o(\tilde{t}) = E_+^o(\tilde{t}) - E_-^o(\tilde{t}) &\approx \hbar\alpha\sqrt{\tilde{t}-\tilde{t}_c} \;; \qquad |\tilde{t}-\tilde{t}_c| \ll \tilde{t}_c \end{aligned}\right\} \quad (109)$$

ie, that in the vicinity of the zero of $\Delta E_\pm^o(\tilde{t})$ when $\tilde{t} = \tilde{t}_c$, the energy difference increases as a square root — in the complex $\tilde{t}$-plane, $\tilde{t}_c$ is a simple zero of the function $(\Delta E_\pm(\tilde{t}))^2$



Quite generally, as we know, the zero of $\Delta E_\pm(\tilde{t})$ is at a complex value - since it is a square root singularity we expect another zero at $\tilde{t}_c^*$, as shown at left.



In the same way as already discussed in section C, we can calculate the transition rate and transition amplitude between the 2 states by considering the complex time contour shown in Fig (a) above, between times $\tilde{t}_{in}$ and $\tilde{t}_f$. Here we will only consider the "in-out" transition rate, between times $\tilde{t}_i \to -\infty$ and $\tilde{t}_f \to +\infty$. Then the system is assumed to pass by the complex root $\tilde{t}_c$, and the transition probability P_{+-} is

$$P_{+-} = |\langle \underline{u}_-(\infty)|\underline{u}_+(-\infty)\rangle|^2 = e^{-\frac{4}{\hbar\varepsilon}\left|\mathrm{Im}\int_0^{\tilde{t}_c} d\tilde{t}\, B_o(\tilde{t})\right|}\, e^{-\Gamma_B} \quad (110)$$

The first term here is the analytic continuation of the dynamic phase in (108); and the 2nd term Γ_B in the exponent is the analytic continuation of the Berry phase ϕ_B:

$$\Gamma_B = 2\, \mathrm{Im} \int_0^{\tilde{t}_c} d\tilde{t}\; \dot{\Phi}_0(\tilde{t}) \cos\Theta_0(\tilde{t}) \qquad (111)$$

Since this integral is just that shown in the Fig. (a) on the last page, taken on each side of the branch cut, we can also rewrite it by simply including the branch cut in the integral – this branch cut extends to the other complex zero of $(\Delta E^0_\pm(\tilde{t}))^2$, at $\tilde{t}_c^*$; see Fig (b). We then can write (111) as an integral over the circuit $\mathcal{C}$ in Fig (b):

$$\left.\begin{aligned} \Gamma_B &= \mathrm{Im} \oint^{\mathcal{C}} d\tilde{t}\; \dot{\Phi}_0(\tilde{t}) \cos\Theta_0(\tilde{t}) \\ &\equiv \mathrm{Im} \oint^{\mathcal{C}} d\Phi_0(\tilde{t}) \cos\Theta_0(\tilde{t}) \end{aligned}\right\} \qquad (112)$$

Now these results are very suggestive. They indicate, first of all, that the results (111) and (112) (which are just the analytic continuation of the Berry phase (106)), written in the form of a closed circuit integral around $\mathcal{C}$, can be thought of as a closed path integral in a complex Hamiltonian space. Indeed, let us now write (106) in the form

$$\phi_B^{\mathcal{C}} = \oint_{\mathcal{C}} d\tilde{t}\; \underline{A} \cdot \dot{\underline{\hat{l}}}_0(\tilde{t}) \equiv \oint_{\mathcal{C}} d\,\underline{\hat{l}}_0 \cdot \underline{A} \qquad (113)$$

where the integral is now in complex time, and the vector potential is that appropriate to the present Hamiltonian, ie.,

$$(\nabla \times \underline{A}) \cdot \underline{\hat{l}}_0 = 1 \qquad (114)$$

(compare (58)).

The second thing that these results suggest is that a re-examination of the details of the path integral formulation of this problem may be interesting, even for a spin-½ system (for details, see Appendix). The discussion of the adiabatic limit, and the slow dynamics of a spin-½ system, is not complete – we will fill it out in the next sub-section (D.2.1(b)) when we consider examples.

(iii) FAST DYNAMICS FOR A SPIN-½:

We will only look at this problem using elementary methods. Recall that in time-dependent perturbation theory for a system with a set of discrete stationary states $|\phi_n\rangle$, such that

$$\mathcal{H}_0 |\phi_n\rangle = \epsilon_n |\phi_n\rangle. \qquad (115)$$

we can calculate the wave function: $\quad |\psi(t)\rangle = \sum_m C_m(t) |\phi_m\rangle \quad (116)$

at some time t, assuming that at time $t = -\infty$ the system is in an initial state $|m_0\rangle$, under the action of a time-dependent perturbation $V(t)$, in the form of an expansion in the matrix elements of $V(t)$ between the stationary states $|\phi_m\rangle$, in the form:

$$\left.\begin{aligned} c_n(t) &= -\frac{i}{\hbar}\int_{-\infty}^{t} dt'\, V_{nm_o}(t')\, e^{i/\hbar(\epsilon_n-\epsilon_{m_o})t'} \\ &= -\left[\frac{V_{nm}(t')}{\epsilon_n-\epsilon_{m_o}}\, e^{i/\hbar(\epsilon_n-\epsilon_{m_o})t'}\right]_{-\infty}^{t} + \int_{-\infty}^{t} dt'\, \frac{\partial V_{nm_o}(t')}{\partial t'}\, \frac{e^{i/\hbar(\epsilon_n-\epsilon_{m_o})t'}}{\epsilon_n-\epsilon_{m_o}} \end{aligned}\right\} \quad (117)$$

Typically one imagines switching on the perturbation, so that as $t\to-\infty$ then $V(t)\to 0$. Then we can write

$$c_n(t) = \bar{c}_n(t) + a_n(t) \quad (118)$$

where the $\bar{c}_n(t)$ are just the changes arising from the time-independent part of $V(t)$, and the $a_n(t)$ from the time-dependent part:

$$\left.\begin{aligned} \bar{c}_n(t) &= -\frac{V_{nm_o}(t)}{\epsilon_n-\epsilon_{m_o}}\, e^{i/\hbar(\epsilon_n-\epsilon_{m_o})t} \\ a_n(t) &= \int_{-\infty}^{t} dt'\, \frac{\partial}{\partial t'} V_{nm_o}(t')\, \frac{e^{i/\hbar(\epsilon_n-\epsilon_{m_o})t'}}{\epsilon_n-\epsilon_{m_o}} \end{aligned}\right\} \quad (119)$$

These results are only valid if $V_{nm_o}(t) \ll |\epsilon_n-\epsilon_{m_o}|$. However if the perturbation is FAST, occurring over a time period Δt such that

$$|(\epsilon_n-\epsilon_{m_o})\Delta t| \ll 1 \quad (120)$$

then we can also get out an approximation, the "sudden approximation"; in what follows we only look at the first term in this, where we assume that the time Δt is so short that we can treat it as a delta function. It then follows that the system state is completely unchanged during the time that $V(t)$ acts, no matter how large it may be. What subsequently happens depends on the form of $V(t)$; if we assume that after the sudden change, $V(t)\neq 0$ (and in general large), then the eigenstates of the system will be different. Writing

$$\left.\begin{aligned} \mathcal{H} = \mathcal{H}_o + V(t) &\xrightarrow{\text{after change}} \mathcal{H}_o + V^0 \\ (\mathcal{H}_o+V^0)\,|\psi_\alpha\rangle &= E_\alpha\,|\psi_\alpha\rangle \end{aligned}\right\} \quad (121)$$

then we see that the state after the perturbation is given by

$$|\Psi(t)\rangle = \sum_\alpha \langle\psi_\alpha|\phi_{m_o}\rangle\, e^{i/\hbar E_\alpha(t-t_o)} \quad (122)$$

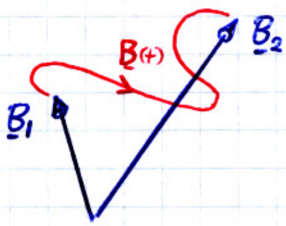
where we assume that the perturbation takes place at time $t=t_o$. If in addition V^0 is small, we have

$$\langle\psi_\alpha|\phi_{m_o}\rangle \longrightarrow \frac{V^0_{\alpha m_o}}{\epsilon_\alpha-\epsilon_{m_o}} \quad (123)$$

in 1st-order perturbation theory (noting that $E_\alpha \to \epsilon_\alpha$ when V^0 is small).
We have recalled these results so as to make clear what is the applicability of these results to the spin problem. To see how this works, it is sufficient to consider the following case: we let

$$\mathcal{H}(t) = \hbar B(t) \cdot \hat{\underline{\tau}} \qquad (124)$$

$$\left.\begin{aligned} B(t) &= \underline{B}_1 + \Delta\underline{B}(t) \\ \Delta B(t) &\xrightarrow[t\to-\infty]{} 0 \; ; \qquad \Delta B(t) \xrightarrow[t\to\infty]{} \underline{B}_2 - \underline{B}_1 \end{aligned}\right\} \qquad (125)$$



The situation is as shown at left; in the period of transition, the trajectory of $\underline{B}(t)$ wanders between $\underline{B}_1$ and $\underline{B}_2$. There are then 2 obvious cases where we can deal with this problem using time-dep. perturbation theory, viz.,

(a) <u>Small perturbation</u>: Suppose $|\underline{B}_2 - \underline{B}_1| \ll |\underline{B}_1|$. Then we write the field in the form

$$B(t) = \underline{B}_1 + \underline{b}(t) = (\underline{B}_1 + \underline{b}_1(t)) + \underline{b}^\perp(t) \qquad (126)$$

where $\underline{b}_1$ is parallel to $\underline{B}_1$. Then the system starts off in a state $|u_1(t)\rangle$ oriented along $\hat{\underline{x}}_1$ (the unit vector in the direction of $\underline{B}_1$). want to know the amplitude for it to be in the state $|u_2(t)\rangle$, oriented along $-\hat{\underline{x}}_1$. From (119) this is just

$$\left.\begin{aligned} c_2(t) &\sim \hbar\int_{-\infty}^{t} dt' \, \frac{\partial}{\partial t} \underline{b}^\perp \cdot \hat{\underline{\tau}} \; \frac{e^{2iB_1 t'}}{2\hbar B_1} \\ &= \int dt' \, e^{2iB_1 t'} \quad b_2^\perp(t') + i b_3^\perp(t')) \end{aligned}\right\} \qquad (127)$$

where we write the components $\underline{b}^\perp = (b_2^\perp, b_3^\perp)$ along axes $\hat{x}_2, \hat{x}_3$ perpendicular to $\hat{\underline{x}}_1$.

(b) <u>Large sudden perturbation</u>: Now we do not assume that $\underline{B}_2 - \underline{B}_1$ is small, but we assume it is sudden - indeed let's assume that

$$B(t) = \underline{B}_1 (1 - \theta(t - t_0)) + \underline{B}_2 \, \theta(t - t_0) \qquad (128)$$

where $\theta(t)$ is the step function. In this case, again assuming that the initial state is oriented along $\underline{B}_1$, we see that the result of the sudden switch from $\underline{B}_1$ to $\underline{B}_2$ will be a