The Geometric Phase

A circuit tracing a closed curve in an abstract space can explain both a curious shift in the wave function of a particle and an apparent rotation of a pendulum's plane of oscillation

by Michael Berry

Take a pencil, lay it on the north pole of a globe and point it in the direction of any of the meridians: the lines of longitude that radiate from the pole. Move the pencil down along the line to the equator and, keeping it perpendicular to the equator, slide it to another line of longitude. Move the pencil back to the north pole along the new meridian, and you will find that although the pencil has been returned to its starting point and at no time was rotated, it no longer points along the original line of longitude.

This simple exercise illustrates how the "parallel transport" of a vector (a quantity that has both length and direction) around a circuit on a curved surface results in an anholonomy: the failure of certain variables describing the system to return to their original values. The anholonomy in the example results from the fact that the pencil was forced to trace out a circuit on the surface of a sphere while remaining parallel to the meridians at all times. It is a purely geometric phenomenon; it does not depend on the energy or mass of the pencil. Moreover, it does not depend on the pencil's initial direction. The extent of the anholonomy depends solely on the area and curvature of the surface enclosed by the circuit.

In 1983 I found that a similar geometric effect exists in the quantum waves that describe matter and its interactions on the smallest scales. In this case the anholonomy appears in a system's wave function (the mathematical description of a system's physical state) after the system has been transported around a cyclic circuit on an abstract surface in "parameter space." I call this anholonomy the geometric phase, because it manifests itself specifically as a shift in the wave function's phase: a quantity that describes where the wave function is in its oscillatory cycle at any given time and place.

It so happens that the geometric phase provides an elegant explanation of various quantum-mechanical phenomena in systems whose environment undergoes a cyclic change: neutrons that pass through a helical magnetic field, polarized light in a coiled optic fiber and charged particles circling an isolated magnetic field. Perhaps more surprising is the fact that the geometric phase can also be generalized to applications in classical physics. Among other things, it offers a new way to describe the behavior of such textbook objects as pendulums.

I discovered the general applicability of the geometric phase in quantum mechanics while studying stationary quantum states, which can be adopted by microscopic systems in unchanging environments. An isolated hydrogen atom provides an example of a stationary quantum state, since the atom's single electron moves in the unchanging electric field of its nucleus. In such a state (which is labeled by a particular set of quantum numbers) measurements of the atom will yield the same result at any time, except for inconsequential shifts in the phase of the wave function describing the system.

Such phase shifts are the result of the dynamical phase inherent in any wave—quantum or classical. Dynamical phase is best understood by considering a familiar example: the traveling wave produced when one jiggles a long extended rope that is held fixed at one end. A series of photographs of the wave would show that those points along the rope that were at the wave's crests in one picture would not be at the crests in another (unless the pictures happened to be synchronized with the wave). In other words, the phase of the wave changes from picture to picture. The rate at which a wave's phase changes in this way is equal to the wave's instantaneous frequency, which for a stationary quantum state is proportional to the state's energy. Because the dynamical phase does not in any way affect the energy or the spatial extent of a quantum system's wave function, it does not influence the system when it is in a stationary state.

The study of stationary states constitutes quantum statics. Statics, however, accounts for only part of both quantum and classical physics. The other part is dynamics, which deals with changing forces and transitions between different stationary states. The area that particularly interested me lies at the border between statics and dynamics; I was studying the effects on a system of very slow changes in its environment. These slow environmental changes, called adiabatic changes, are the subject of a major theorem that was originally conceived outside a formal quantum-mechanical framework by Albert Einstein and Paul Ehrenfest in 1911 and rigorously proved within the framework by Max Born and Vladimir A. Fok in 1927.

According to the quantum adiabatic theorem, a system initially in a stationary state that is labeled by a certain set of quantum numbers will remain in a stationary state that is labeled by the same set of quantum numbers even though its environment may be slowly changing. The power of the theorem lies in the fact that the initial and final environments—and hence the actual form of the corresponding stationary states—can be rather different: the adiabatic condition stipulates only that the environmental change is slow, not that it is

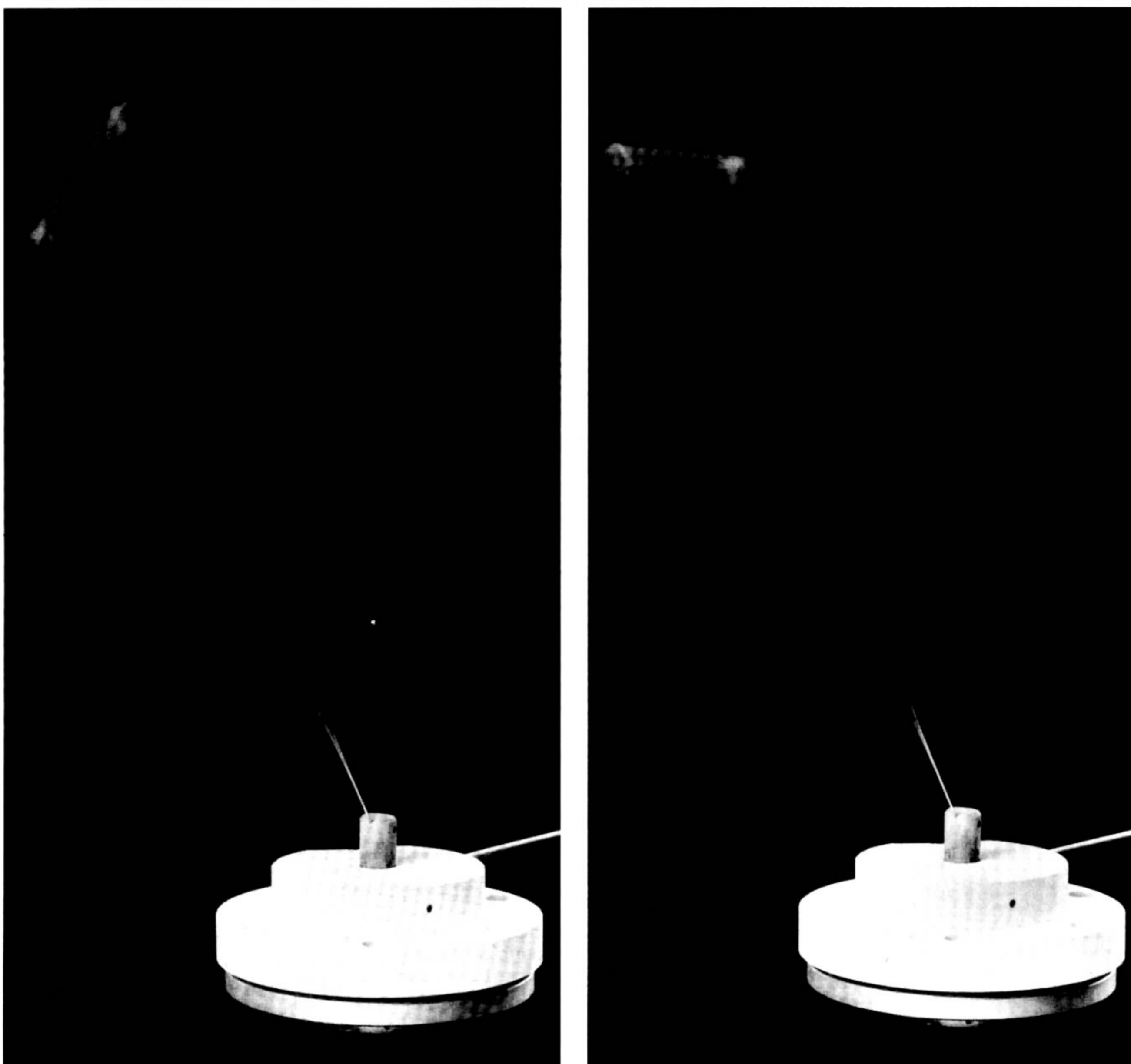
MICHAEL BERRY is professor of physics at the University of Bristol, where he studies geometric aspects of waves—in particular those aspects involving connections between quantum and classical mechanics. He holds degrees from the University of St. Andrews and the University of Exeter. Berry has been at Bristol since 1965.

small. (If the change is not slow, the theorem does not apply and the system will make transitions to states labeled by other quantum numbers.)

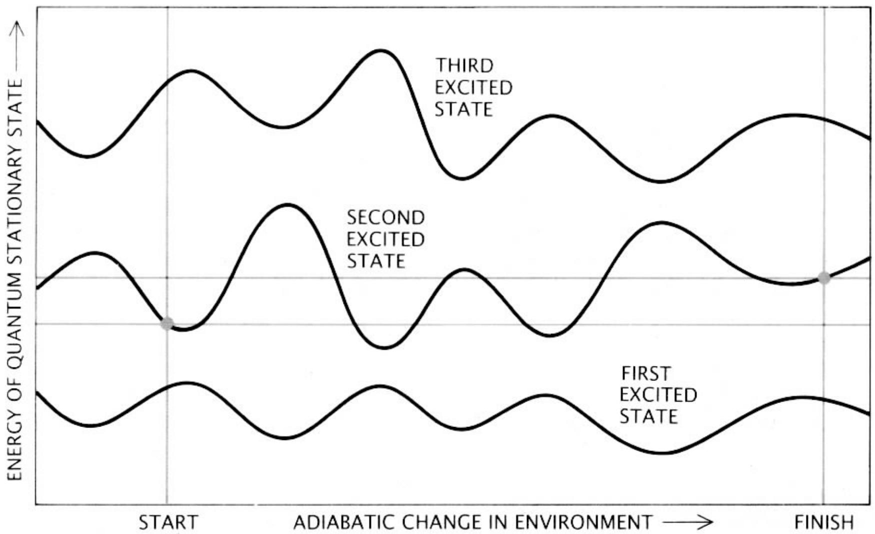
Perhaps the most useful application of the adiabatic theorem is in approximating the quantum states of a molecule, which is a collection of constantly moving electrons and nuclei. An exact solution of the equation that determines the quantum mechanics of even the simplest molecule (the hydrogen molecule in a charged form that has two protons and one electron) has eluded physicists. Yet because nuclei are several thousand times more massive than electrons, they can be considered to move much more slowly than the electrons. Since the nuclei constitute the electrons' "environment," the electron states can be said to evolve adiabatically as the nuclei move. The continuous motion of the nuclei can therefore be broken down into a sequence of "frozen" configurations, each of which has quantum states given by the electrons' corresponding stationary states.

Since a quantum system in a slowly changing environment remains in a stationary state, it may seem as though an adiabatic change is really no change at all. That, at any rate, was the prevalent opinion among physicists for many years, and because of it a curious and subtle effect of an adiabatic change on the phase of a wave function was overlooked.

Consider the phase of the wave function of a quantum system that has undergone a cyclic adiabatic change—one that leaves the environment in the same condition as before the change. Although the initial and final states of the system will be the same, the phases of the initial and final wave functions will be dif-



ANHOLONOMY—the phenomenon underlying the geometric phase—is shown by a device that Moshe Kugler and Shmuel Shtrikman of the Weizmann Institute of Science in Israel have built. An anholonomy is a geometric effect in which variables describing a system do not return to their original values after the system completes a revolution. In the device a wire in a base is set vibrating in a plane (*left*) and the base is turned. After a revolution (*right*) the wire no longer vibrates in the plane.



ADIABATIC CHANGES are changes in a system's environment that occur slowly enough for the system to stay continuously in equilibrium with its environment. A system normally adopts a so-called stationary state (which is labeled by a set of quantum numbers) when its environment is static. Yet according to the quantum adiabatic theorem, a system in a stationary state can remain in a stationary state labeled by the same set of quantum numbers even if the environment changes—provided the changes are adiabatic. The theorem applies even if the system's final environment and energy are very different from its initial environment and energy.

ferent owing to the time-dependent dynamical phase of the wave function. This phase difference would exist even if the environment did not change; it simply reflects the time it took the system to complete the cycle.

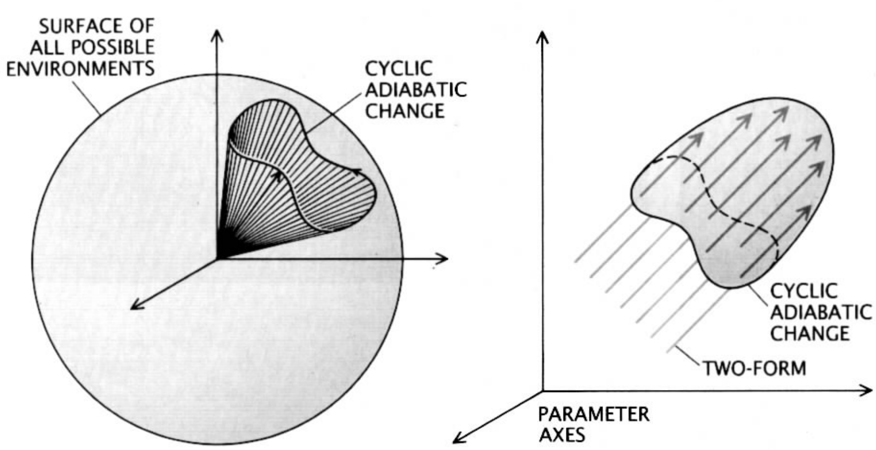
This much was well known. I was able to show that any cyclic adiabatic change can shift the phase of the wave function in another, rather surprising way and derived a formula for the new phase shift from quantum physics. The formula is best understood by visualizing the slow changes in the environment as a closed circuit in an abstract frame of reference whose axes are parameters: physical variables that describe the system's environment. An analogy to the pencil-and-globe exercise becomes apparent: the phase shift can be seen as the result of an anholonomy that arises whenever the system is made to complete a circuit on a curved surface in the parameter space.

Indeed, as Barry M. Simon of the California Institute of Technology recognized, the mathematics for the parallel transport of a vector around a circuit on a curved surface yields (when properly generalized) the same answer as the formula for the phase component that I obtained from quantum physics. As in the pencil-and-globe example, the phase shift can be calculated from the area and curvature of the surface enclosed by the circuit.

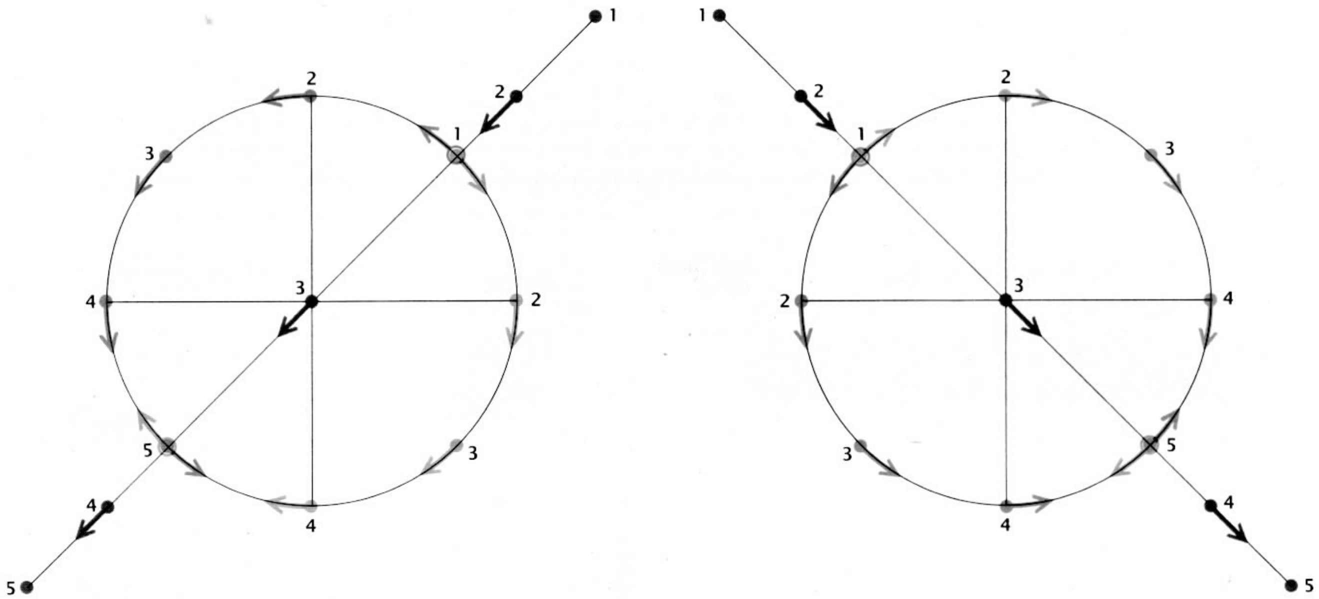
Because such a phase shift depends only on the geometry of the circuit in parameter space, I call it the geometric phase. It is independent of the time it takes the system to complete the circuit (provided that the process occurs slowly enough to constitute an adiabatic change), but it is sensitive to the shape of the circuit and the system's initial quantum state. The geometric phase can therefore be regarded as the best answer the system can offer to the question "What path in parameter space did the system take?" In this sense it is a kind of quantum "memory": it contains information about the past environments of the system.

Since I established the theoretical applicability of geometric phase in any cyclic adiabatic change, it has been calculated for a variety of systems cycling through a variety of parameter-space circuits. The phenomenon has also been measured experimentally in quantum spins that have been "turned" slowly. The quantum spin of a particle can be pictured roughly as the spin of a tiny top about an axis. The stationary spin states of the particle are characterized by a quantum number that gives the value of its angular momentum (which is a vector quantity) as measured along a direction determined by the particular symmetry of its environment. Such a symmetry direction is given, for example, by the direction of a magnetic field for particles that are susceptible to magnetic forces. If the symmetry direction is slowly changed, the adiabatic theorem ensures that the spin of the particle turns with the symmetry direction, preserving the particle's angular-momentum component along the symmetry direction and therefore its spin quantum number.

A symmetry direction can be repre-



GEOMETRIC PHASE of a quantum system whose environment has undergone a cyclic adiabatic change can be derived by plotting all possible environments of the system in a frame of reference whose axes are parameters: the physical variables that describe the environment. A cyclic adiabatic change is then represented as a closed curve in the "parameter space." In the simplest case the geometric phase is given in terms of the area of any surface the curve encloses. If the surface is spherical (*left*), the area is equivalent to the solid angle subtended by the curve. The geometric phase can be more readily generalized to parameter spaces with more than three dimensions if it is expressed in terms of a mathematical quantity called a two-form (*right*). A two-form can be thought of as representing the flux, or flow, of a quantity through space. The geometric phase can then be calculated by integrating, or summing, the two-form over any surface that "catches" all the two-form flux through the circuit.



SUPERPOSITION of two oppositely directed circular motions can result in linear motion. Summing the coordinates of the red and the blue points, which are tracing concentric circles of equal radius, yields the coordinates of a third point (*green*) that slides back and forth along a line. The direction of the line depends on the relative phases of the two circling points. If the two points start their motions in the upper right-hand quadrant of the coordinate system (*left*), the line will be tilted at an angle of 45 degrees. If they start in the upper left-hand quadrant (*right*), however, the angle will be 135 degrees. A similar principle explains how two superposed states of circularly polarized light rotating in opposite senses can result in linearly polarized light. As with the moving points, the relative phases of the states determine the light's direction of polarization.

sented by a vector of unit length and fixed origin. Since a symmetry direction is an arbitrary direction, the tips of all possible symmetry-direction vectors lie on a sphere of unit radius whose center is the origin of the vectors. The surface of this unit sphere represents the parameter space for turned spins, since the symmetry direction determines the environment in which a particle's spin is measured. Any sequence of changes in the symmetry direction can therefore be charted as a curve on the surface of the sphere. A closed curve, of course, would mean that the changes are cyclic. Employing quantum theory, I showed that in tracing such a curve the particle acquires a geometric phase given by the product of its spin-state quantum number and the solid angle enclosed by the curve on the parameter-space sphere.

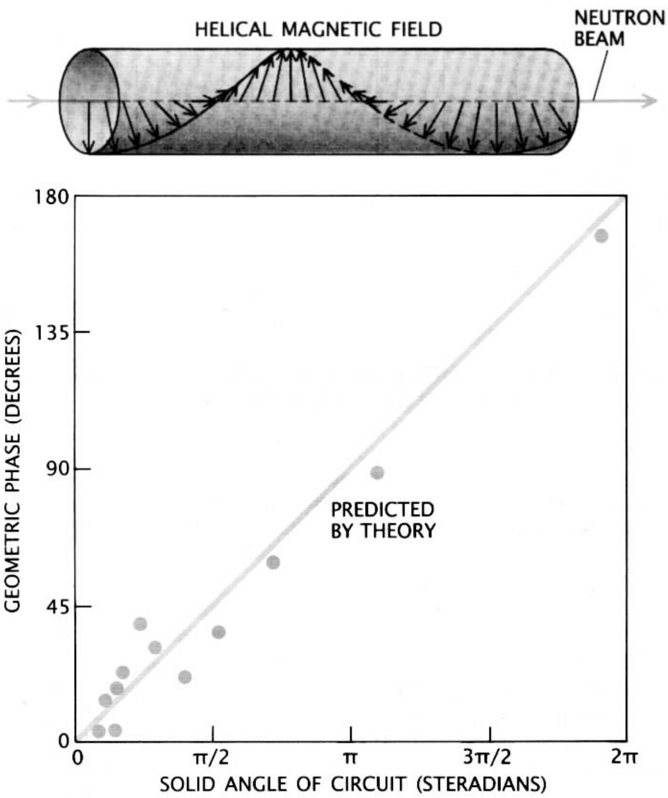
How can one measure the geometric phase of turned microscopic particles? In spite of its fundamental nature, the phase of a quantum wave cannot be detected directly; it becomes measurable only when two or more quantum waves are brought together to produce so-called interference patterns. When two waves are summed, the amplitude of the resulting wave is the sum of the amplitudes of the component waves wherever a crest of one wave coincides with a crest of the other and a trough coincides with a trough. But wherever crests coincide with troughs, the amplitude of the resulting wave is the difference between the amplitudes of the two component waves. Hence the pattern of amplitudes of the resulting wave—the interference pattern—reveals the relative phases of the component waves. (The phase difference of two waves is the fraction of a cycle through which one wave must evolve for its crests and troughs to coincide with those of the other. Fractions of a cycle are generally expressed in angular units, such as degrees or radians, where one complete cycle equals 360 degrees, or 2π radians.)

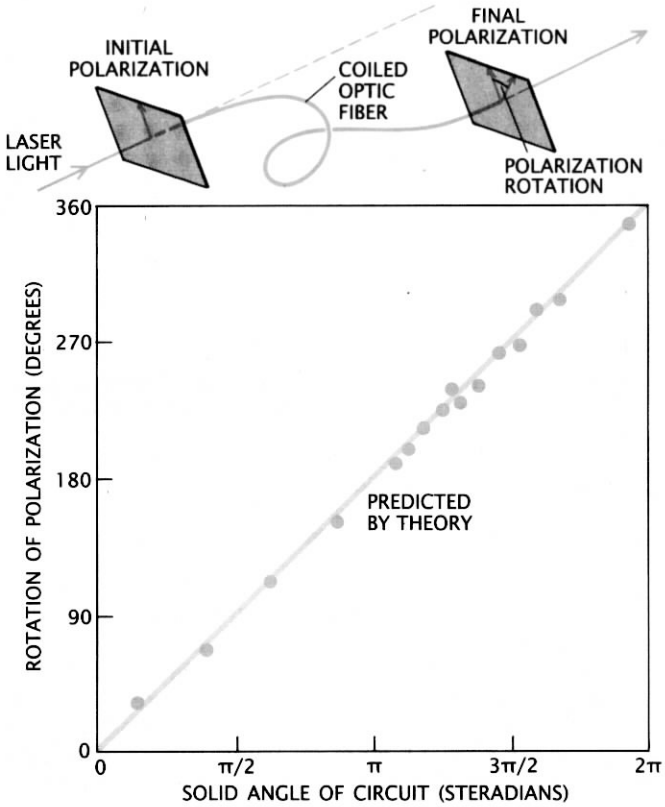
Clearly in order to measure the geometric phase of a turned particle's wave function the particle must first be "added" to a second particle to produce measurable interference patterns. In principle it is possible to do so by splitting a beam of particles that are all in the same spin state, turning the spins of one of the split beams and then recombining the beams. This can sometimes be done in practice, but the experiments have often been difficult to carry out.

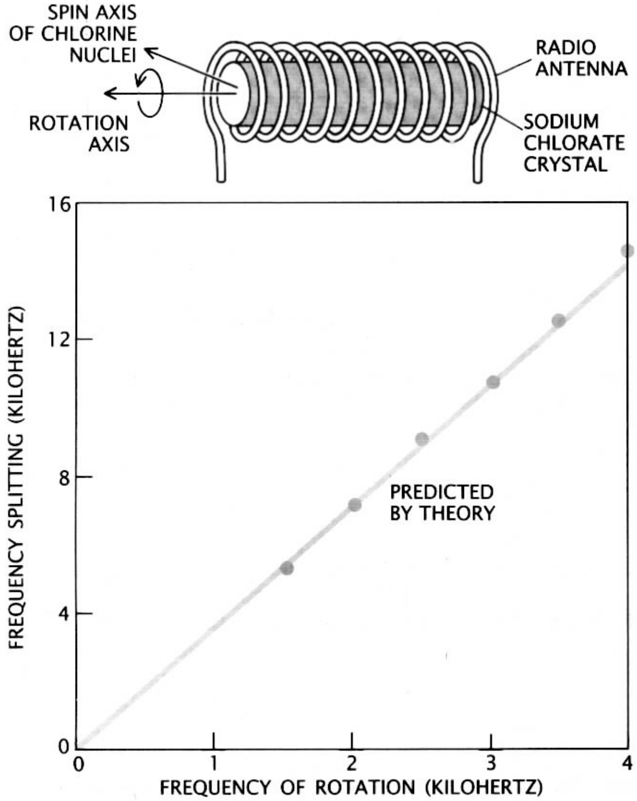
More commonly the initial beam consists of particles in a superposition of different stationary spin states, each labeled by a different spin-state quantum number. The superposed states combine with one another in a way that depends on the relative phases of their respective wave functions. Because the geometric phase of the wave functions depends on their respective spin-state quantum numbers, the phases of the constituent wave functions are shifted differently as a particle's spin is being turned, changing the way the states combine to form the superposed state. Such changes in the form of the superposed state are generally easier to detect than changes in the interference pattern created by combining turned and unturned single-state particle beams.

The solid-angle formula for the geometric phase of turned spins has been confirmed for several different types of particles. T. Bitter of the University of Heidelberg and D. Dubbers of the Laue-Langevin Institute in Grenoble worked with neutrons, whose spin and magnetic-moment directions coincide. The neutron's magnetic moment makes it possible to turn its spin by altering the direction of a magnetic field, since the field lines provide the symmetry direction. In their experiment Bitter and Dubbers changed the symmetry direction cyclically by passing a beam of neutrons through a helical magnetic field produced by a twisted current-carrying coil. The solid angle subtended by the closed curve in the corresponding parameter space could readily be varied by altering the strength of another magnetic field along the beam's axis.

Although a photon also has spin, it does not have a magnetic moment with which its spin can be turned. Raymond Y. Chiao of the University of California at Berkeley, Yong-shi Wu of the University of Utah and Akira Tomita of the AT&T Bell Laboratories nonetheless devised and carried out an experiment in which the spin of photons was turned. They relied on the fact that a photon's spin vector points either along the direction in which it is traveling or in the opposite direction. Hence a photon's spin can be turned merely by changing its direction of travel. By confining a beam of laser light in a coiled optical fiber, Chiao and his colleagues were able to turn the photons' spin; by ensuring that the







THREE EXPERIMENTS in which the spin state of particles is cyclically "turned" confirm the reality of the quantum geometric phase. T. Bitter of the University of Heidelberg and D. Dubbers of the Laue-Langevin Institute in Grenoble took advantage of the fact that a neutron's spin axis coincides with its magnetic moment to turn a beam of neutrons by passing it through a helical magnetic field whose pitch could be varied (*top left*). The geometric phase was measured as a shift in the spin axis of the neutrons. Raymond Y. Chiao of the University of California at Berkeley, Yong-shi Wu of the University of Utah and Akira Tomita of the AT&T Bell Laboratories turned two superposed spin states of photons by shining linearly polarized light through a coiled optic fiber (*top right*). The observed rotation of the light's direction of polarization equals the predicted geometric phase. In an experiment carried out by Robert Tycko of Bell Laboratories the spins of excited chlorine nuclei were turned simply by rotating a crystal of sodium chlorate on an axis that was different from the spin axis of the nuclei (*bottom left*). The nuclei accumulated a geometric phase that was detectable as a frequency shift in the radio signals they emitted.

two ends of the fiber were parallel, the investigators made the process cyclic.

They also took advantage of the fact that linearly polarized light (light whose associated electric field vibrates in a single direction) consists of photons in which the two possible spin states are superposed. The direction of the polarization is given by the relative phases of the two spin states [*see illustration on page 49*]. As a consequence, any change in the relative phases that arises as the two different spin states acquire different geometric phases can be directly observed as a rotation of the light's direction of polarization. (Such a rotation was first observed in 1984 by J. Neil Ross of the Central Electricity Generating Board Laboratory in Leatherhead, England, but he did not attribute it to the geometric phase.)

The rotation that Chiao and his colleagues observed can be explained equally well in classical terms. It can be understood as the result of a parallel transport of the light's electric-field vector along the coiling fiber. For light in a coiled fiber, then, the quantum anholonomy of phase is equivalent to the classical anholonomy of parallel transport of polarization, or, as Chiao and Wu express it: "We would rather think of these effects as topological features... that originate at the quantum level but survive... into the classical level." As several other investigators have pointed out, the solid-angle result in this case can also be derived without resorting to quantum mechanics, from Maxwell's equations of classical electromagnetism.

The work of Robert Tycko of Bell Laboratories provides a final example of turned spins. Tycko excited chlorine nuclei contained in a crystal of sodium chlorate into a superposition of states by exposing them to a pulse of radio waves. Since the spins of the nuclei are aligned with the crystal's axis of symmetry, he was able to turn the spins of the excited nuclei by rotating the crystal about an axis different from its symmetry axis. Consequently the phase difference between the nuclei's component spin states increased according to the solid-angle formula. The effect of repeated rotations was to increase the phase difference continually, which Tycko detected as a splitting in the frequency of a radio-signal response emitted later by the nuclei.

In all the turned-spin experiments I have mentioned, it has been assumed that the environmental parameters governing a system can be determined (at least in principle) to arbitrarily high precision and that the environment remains unaffected by any phase changes it induces in the system. Actually neither supposition is justified. The first one is invalid because the parameters, being physical variables, are themselves subject to the laws of quantum mechanics, which stipulate an inherent uncertainty in their measurement. The second supposition is also invalid because in physics there is no such thing as a unilateral action. For these reasons, what I have referred to until now as the "quantum system" should strictly speaking include the environment: the laboratory apparatus in which the turned-spin experiments were done. Yet because a full quantum-mechanical description of the environment is indescribably complicated, it is usually neglected.

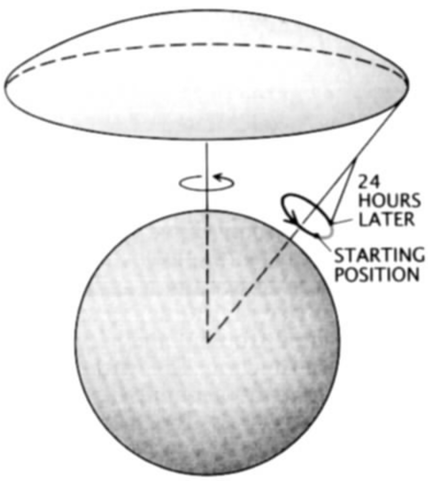
Nevertheless, one can predict a curious effect a quantum system's geometric phase will have on the wave function of its environment. It so happens that the wave function of any state of the "total" system (the product of the wave functions of both the quantum system and its environment) must be a single-valued function: it must have only one unique value—including phase—for any given set of parameters. This is mathematically possible only if, during any circuit of the quantum system in parameter space, the wave function of the environment acquires a compensating phase shift equal to that of the system but of opposite sign.

It might be thought this equal but opposite phase shift is a mathematical artifact arising from the separation of the quantum system and the environment, but in fact it can give rise to observable physical effects. Such effects occur, for example, in molecules in which the electrons constitute the quantum system and the nuclei constitute the environment. In 1979 C. Alden Mead and Donald G. Truhlar of the University of Minnesota (who laid the groundwork for much of the subsequent work on the geometric phase) pointed out that changes in the wave function for the electrons (the system) should affect the wave functions describing the motion of the nuclei (the environment), thereby altering the energies corresponding to vibrations and rotations of the molecule. Such changes are reflected in the energy of the photons emitted or absorbed by a molecule and can be detected by spectroscopy.

An experiment carried out by Guy Delacrétaz and Ludger Wöste of the Swiss Federal Institute of Technology, Edward R. Grant and Josef W. Zwanziger of Cornell University and Robert L. Whetten of the University of California at Los Angeles detected such spectroscopic changes in a molecule composed of three sodium atoms. The sodium nuclei in the molecule undergo a cyclic motion (called pseudorotation) that causes the lowest electron-energy state to acquire a striking geometric phase of 180 degrees: crests have become troughs and vice versa in its wave function. The workers discovered that the geometric phase of the electrons' wave function in turn affects the nuclei's wave function, changing the observed pseudorotation energy levels of the nuclei. The changes were consistent with the predictions of an extended quantum-mechanical analysis of geometric phase.

The geometric phase of a system undergoing a cyclic adiabatic change can be stated most elegantly when it is expressed in terms of a mathematical quantity called a two-form, which represents the flux, or flow, of a quantity through a unit area. The geometric phase is then calculated merely by integrating, or summing, the two-form over any surface spanning the system's circuit in parameter space—any surface that "catches" all the two-form flowing through the circuit. Such a powerful mathematical formulation conjures up the image of the two-form lurking in parameter space, ghostly and hidden until actualized by a quantum system completing a cycle in parameter space. An effect predicted in 1959 by Yakir Aharonov of the University of South Carolina and David Bohm of the University of London (both of whom were then working at the University of Bristol) can be explained precisely in terms of such a geometric-phase two-form.

The Aharonov-Bohm effect is a shift in the phase of a charged particle's wave function produced by transporting the particle around isolated magnetic field lines. The effect was confirmed experimentally in 1960 by Robert G. Chambers of Bristol. In this case the parameter space through which the particle moves is not defined by abstract variables such as symmetry directions; it is the ordinary space through which the particle makes its circuit, described in terms of familiar coordinates (measuring length, width and height). Similarly, the phase two-form is not just a convenient mathematical construct; it is the magnetic field multiplied by the charge of the particle and divided by Planck's con-



ANGULAR SHIFT in the position of the bob of an earth-based pendulum is an example of the geometric phase in classical physics, as described by John H. Hannay of the University of Bristol. One might think that a pendulum bob moving in a circle at one revolution per second would, after 24 hours, return to exactly the same position in space from which it was set in motion. But in fact the bob's position will be shifted in relation to its initial position by an angle (*red*), known as Hannay's angle, equal to the solid angle subtended by the pendulum's axis of revolution (*blue*). Such a pendulum was employed by the French physicist Jean B. L. Foucault in 1851 to demonstrate convincingly the rotation of the earth. Foucault's pendulum, however, swung to and fro rather than in a circle, and the geometric phase manifested itself as a rotation of the bob's plane of oscillation.

stant (6.626×10^{-34} joule-second). Expressed in this way, the phase of the particle passing around the magnetic field depends on the magnetic field flux—a quantity that fits squarely in classical physics.

What does not fit squarely in classical physics, however, is the fact that the charged particle's phase is affected by the magnetic field, in spite of the fact that the particle never crosses a field line. In classical physics charged particles experience forces only when they come in contact with electric or magnetic fields. Yet a charged particle in an Aharonov-Bohm experiment is affected by a magnetic field even though it is kept separate from the field! Physicists say that the field appears to influence the charged particle nonlocally. Unlike the rotation of the plane of polarization in the experiment of Chiao and his colleagues, the Aharonov-Bohm effect cannot be explained in terms of classical physics.

A geometric-phase two-form lends itself to describing quantum-mechanical phenomena that are foreign to our everyday experience, but it can also be generalized to describe even the familiar mechanics of springs and pendulums. John H. Hannay of Bristol has worked out the classical analogue of the quantum-mechanical geometric phase. He began by considering macroscopic systems of oscillating bodies whose configuration at any time is given by one or more angle variables. The environment of the oscillations is made to change slowly, but the process begins and ends at the same set of parameters. After the cycle the oscillations have the same amplitude as they had originally, but the angles have changed.

Hannay realized that the angular shifts can be divided into dynamical and geometric parts, just as the phase shift of a quantum-mechanical system can be. The dynamical part is what would be calculated on the assumption that the angle increases at a rate corresponding to the instantaneous frequency of oscillation. His achievement was to identify the geometric contribution, now called Hannay's angle, and to derive a formula by which it can be calculated as the flux of a two-form through a closed circuit in parameter space. (The analogy with quantum mechanics is not complete, however, because classical motions are often not oscillatory but chaotic. For such systems no angle variables can be defined, and there are no Hannay angles.)

In one of Hannay's examples a bead is imagined as sliding at a constant speed and without friction on a noncircular loop of wire as the loop is slowly rotated once in its own plane. In this case the angle variable is the bead's distance around the loop as measured from a given point on the loop. Hannay's angle gives the position of the bead after the loop's rotation in relation to where the bead would be if the loop had been held stationary. The angle is a purely geometric combination of the perimeter of the loop and the area it encloses; it is large for a long thin loop and vanishes for a circular one.

Another of Hannay's examples gives the classical analogue of the quantum geometric phase for slowly turned spins. Consider a pendulum bob moving in a circle. In this case gravity determines the symmetry direction, namely a vertical line passing through the center of the earth. As the earth rotates, the symmetry direction turns in space (unless the experiment is done at one of the poles), so that after one day the position of the pendulum bob in its circular orbit will be shifted by an angle—Hannay's angle—equal to the solid angle subtended by the symmetry direction.

Such a shift in the position of the pendulum bob is more conspicuous if the bob moves to and fro rather than in a circle. The bob's to-and-fro motion can then be regarded as a superposition of two circular motions in opposite directions (just as linearly polarized light can be regarded as the superposition of two circularly polarized states of light). After a day of swinging, the pendulum's two circular motions will have acquired opposite angle shifts, which manifest themselves as a rotation of the plane in which the bob swings.

What I have just described is the pendulum with which the French physicist Jean B. L. Foucault demonstrated, in 1851, the rotation of the earth. The well-known phenomenon of the rotation of a pendulum's plane of oscillation, which is a popular exhibit in many science museums throughout the world, is thus a special case of Hannay's angle, which in turn is the classical analogue of the quantum geometric phase. It too can be explained as the result of parallel transport, in this case of the pendulum's swing plane by the earth's rotation.

By returning to the parallel transport with which this article began, one can say I have completed a cycle. Yet like the phase of a system undergoing an adiabatic cycle, the end is different from the beginning. The parallel transport that initially illustrated an abstract concept now appears in tangible systems whose behavior is governed by the laws of physics for slow environmental changes. Geometric anholonomy has turned into dynamical anholonomy.

FURTHER READING

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