SOME MOCK EXAM QUESTIONS for PHYS 501 (2010)

You will also find the 2009 mock exam questions useful to look at (where they are relevant). The following questions emphasize somewhat the material not covered in the 2009 mock exam, or in the 2009 exam.

REMARKS on the EXAM

The exam will last 2 hrs and 30 mins. The only material allowed into the exam will be pens, pencils, and erasers. No notes of any kind are permitted, nor any calculators.

There will be 2 sections. Students should answer THREE QUESTIONS ONLY from section A, and TWO QUESTIONS ONLY from section B. No extra marks will be given for extra questions answered. The questions in section A should take roughly 15-20 minutes to answer, and the questions in section B roughly 45-50 minutes to answer.

MOCK QUESTIONS

MOCK SECTION A QUESTIONS

A1: Consider a 2-dimensional classical harmonic oscillator, with Hamiltonian

$$\mathcal{H} \;=\; \frac{1}{2}\left[\left(\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}\right)+gx_1x_2+(m_1\omega_1x_1^2+m_2\omega_2x_2^2)\right] \tag{1}$$

Find the normal modes q_1, q_2 of this system, and the natural frequencies Ω_1, Ω_2.

Now assume the system is quantized, and that $\Omega_2 = 2\Omega_1$. What is the density of states of the system as a function of energy?

A2: Suppose we have a Lagrangian of form

$$L(Q,\dot{Q};t) \;=\; [\alpha(t)\dot{Q}^2+\beta(t)Q\dot{Q}+\gamma(t)Q^2] \tag{2}$$

where Q is a generalised coordinate. Suppose we can write the classical action for the a general path $Q(t)$ of the system as an expansion of form $\mathcal{S}[Q] = \mathcal{S}[Q_c(t)+q(t)]$, where $Q_c(t)$ is the classical trajectory of the system, and $q(t)$ describes small fluctuations about this trajectory. Assuming that an expansion to lowest quadratic order in fluctuations is enough, we can write the 1-particle propagator for the system in the form

$$G(Q_2,Q_1;t) \;=\; A(t)e^{\frac{i}{\hbar}\mathcal{S}_c(Q_2,Q_1)} \tag{3}$$

where $\mathcal{S}_c = \mathcal{S}[Q_c]$, and the prefactor $A(t)$ can be evaluated as a path integral over the fluctuation coordinate $q(t)$. Find an expression for this path integral.

A3: The classical action $\mathcal{S}$ for a classical system can be written as $\mathcal{S} = \int(PdQ-\mathcal{H}dt)$, where P, Q are generalised momenta and coordinate variables, and $\mathcal{H}$ is the Hamiltonian. Derive the

Hamilton-Jacobi equation for the action, in the form

$$\frac{\partial \mathcal{S}}{\partial t} + \mathcal{H}(\frac{\partial \mathcal{S}}{\partial Q}, Q; t) = 0 \tag{4}$$

and show that $\partial \mathcal{S}/\partial Q = P$. From this write down the Hamilton-Jacobi equation for a particle in a 2-d potential $V(\mathbf{r})$, and draw a picture which shows schematically the contours of constant action and the particle trajectories in this potential.

A4: A system of two spin-1/2 particles is in an initial pure correlated state of form

$$|\Psi\rangle \; = \; a|\uparrow\uparrow\rangle + be^{i\theta}|\uparrow\downarrow\rangle + ce^{i\phi}|\downarrow\downarrow\rangle \tag{5}$$

where a, b, c are real and $a^2 + b^2 + c^2 = 1$. Write down the density matrix for this system in the 4×4 state basis, showing how you have labelled the states in the matrix. Now, suppose we have no knowledge of the state of the 2nd spin, and must average over it. What is the reduced density matrix for the first spin?

A5: A spin-1/2 system is in a state $|\uparrow\rangle$ in a large field $\mathbf{B}_o = \hat{z}B_o$, when it is subject to a field pulse of form $b(t) = \hat{x}b_o\delta(t)$, with $b_o \ll B_o$. Find the subsequent motion of the spin, using first-order time-dependent perturbation theory in the sudden approximation to find its state immediately after the pulse, and then solving Schrodinger's equation for $t > 0$.

A6: Describe how a Geiger counter is used to detect passing high-energy particles. Suppose the cylinder-shaped counter has a voltage V_o across a length R_o (the internal radius of the cylinder), and an incoming particle of energy E deposits all of its energy into the gas particle of the counter. If the ionization energy of the gas particles is ϵ_o, what is the maximum possible charge pulse generated in the counter by the incoming particle? From the point of view of quantum measurement theory, what do you think is the coordinate of the measuring apparatus?

A7: In 2 dimensions the Hamiltonian for a charged particle in a perpendicular magnetic field is

$$\mathcal{H} \; = \; \frac{1}{2m}(\hat{\mathbf{p}} + e\hat{\mathbf{A}}_o(\mathbf{r}))^2 \tag{6}$$

where we assume that the vector $\nabla \times \mathbf{A}_o(\mathbf{r}) = H_o\hat{z}$.

Suppose a particle of charge q is confined to move on a ring of radius R_o in the xy-plane. Find the eigenvalues and eigenstates of the Schrodinger equation for this particle.

A8: A 2-level system has the Hamiltonian

$$\mathcal{H}(t) \; = \; B_z\hat{\tau}_z + b_x\hat{\tau}_x\theta(t) \tag{7}$$

Suppose the system starts for $t < 0$ in the state $|\uparrow\rangle$; find the solution to the Schrodinger equation for this system for $t > 0$.

A9: Find the *bound state* solutions only (eigenvalues and eigenfunctions) to the Schrodinger equation for a 1-d potential in which $V(x) = -V_o(\delta(x-a) + \delta(x+a)$, where V_o is positive.

A10: Consider a particle of charge q which is forced to move on a sphere of unit radius. Suppose there is a magnetic monopole of strength g at the centre of the sphere, so that the magnetic field is

$$\mathbf{B}(\mathbf{r}) \; = \; \frac{g}{4\pi}\frac{\mathbf{r}}{r^2} \tag{8}$$

as a function of radius vector $\mathbf{r}$. Derive the Dirac quantization condition $gq = nh$, where h is Planck's constant, and n is an integer.

A11: The Holstein-Primakoff representation for spin writes

$$\begin{aligned} \hat{S}_z &= S - b^+b \\ \hat{S}^+ &= (2S)^{1/2}[1 - \frac{1}{2S}b^+b]^{1/2}b \\ \hat{S}^- &= (2S)^{1/2}b^+[1 - \frac{1}{2S}b^+b]^{1/2} \end{aligned} \tag{9}$$

where $\hat{S}^{\pm}$ are spin ladder operators. Show that the operators b, b^+ satisfy the correct bosonic commutation relations. Then, assuming we have a Hamiltonian

$$\mathcal{H} \; = \; K_z\hat{S}_z^2 - \gamma B_o\hat{S}_x \tag{10}$$

find an expansion for this Hamiltonian up to 4th order in the boson operators (assuming that the expectation value $< b^+b > \; \ll 1$.

A12: The Hamiltonian for a spin-1/2 system in an arbitrary time-dependent field $\mathbf{B}(t)$ is $\mathcal{H} = \hbar\mathbf{b}(t)\cdot\boldsymbol{\tau}$, where $\boldsymbol{\tau}$ is a Pauli operator, and $\mathbf{b}(t) = -\gamma\mathbf{B}(t)/2$, where $\mathbf{B}(t)$ is the applied field.

Find the Schrodinger equation for the 2-component spinor wave-function in the case where $b_z(t) = b_o$ is constant in time.