Phys 501: HOMEWORK ASSIGNMENT No (5)

Sunday April 5th 2009

DUE DATE: Wednesday April 15th 2009.
Assignments handed in late may not receive a full mark.

This assignment is not arduous - its main purpose is to make you think about the material that we have done at the end of the course, and see what is important. Note that I will continue to have time reserved for tutorials for anyone who wants to come, right up until the exam - and I will also compose some 'mock' exam questions.

QUESTION (1): BERRY PHASE

The following identities are needed in the demonstration of the result for the Berry phase.

(i) First, for a Hamiltonian $\mathcal{H}(g)$ depending on a parameter g, and having adiabatic eigenvalues $E_n(g)$ and eigenstates $|\psi_n(g)\rangle \equiv |n(g)\rangle$, show that

$$(E_n - E_m)\langle m|\nabla_g n\rangle \;=\; \langle m|\nabla_g \mathcal{H}|n\rangle \tag{1}$$

when $m \neq n$.

(ii) Second, if the Berry phase is defined as $\phi_B^n(\mathcal{C}) = i \oint_{\mathcal{C}} d\mathbf{g} \cdot \langle n(g)|\nabla_g n(g)\rangle$, where the line integral is taken around the circuit $\mathcal{C}$, then show that

$$\begin{aligned} \phi_B^n(\mathcal{C}) &= -Im \oint_{\mathcal{C}} d\mathbf{S} \cdot \nabla_g \times \langle n|\nabla_g n\rangle \\ &= -Im \oint_{\mathcal{C}} d\mathbf{S} \cdot \sum_{m\neq n} \langle \nabla_g n|m\rangle \times \langle m|\nabla_g n\rangle \end{aligned} \tag{2}$$

where the integration is over the surface in parameter space (ie., $\mathbf{g}$-space) enclosed by the closed curve $\mathcal{C}$.

QUESTION (2):

(i) A spin $S = 1$ is oriented at time $t = 0$ in the xz-plane along an angle 45^o between the x and z axes. A magnetic field B_o along the z-axis is applied at $t = 0$. Assuming an electronic g-factor 2, determine the precession period T for the spin. Then, suppose we allow the spin to precess for a time $t = 10T$. What is the final state of the spin wave-function?

(ii) Now suppose we have the same spin in the same initial state as in (i), but we now apply a field of strength B_o at $t = 0$ which is parallel to the spin's initial state. We then move this field very slowly in a circuit around the z-axis, completing 10 full circuits, always keeping the field at an angle of 45^o from the z-axis. What now is the final state of the spin wave-function?