Phys 501: HOMEWORK ASSIGNMENT No (4)

Friday March 20th 2009

DUE DATE: Wednesday April 1st 2009.
Assignments handed in late may not receive a full mark.

QUESTION (1): T-MATRIX and SCATTERING

(i) Suppose we have a T-matrix which we know has a simple pole (ie., of unit residue) at energy $E = -E_o$, and a branch cut of magnitude

$$A(E) = \frac{(N_o/\pi)}{[1 + (E/\Omega_o)^2]}\theta(E) \tag{1}$$

starting from the origin. Find the T-matrix $T(z)$ as a function of the complex energy z, assuming that $|T(z)|$ goes to zero sufficiently rapidly as $|z| \to \infty$. Write down also the form of the Green function $G(z)$ that follows from this.

(ii) Consider again the 1-dimensional "double-barrier" potential of form

$$V(x) \;=\; V_o[\delta(x - a_o/2) + \delta(x + a_o/2)] \tag{2}$$

where each δ-function barrier has the same strength (you also looked at this problem in the last assignment).

Find the T-matrix and the lowest-order Born approximation result for the scattering amplitude off this potential.

QUESTION (2): BOUND STATES and RESONANCE IN 2D

Consider a 2-d "Delta-shell" potential with a δ-function repulsive potential in the centre of it; the form of this is

$$V(r) \;=\; V_o\delta(r - R_o) + U_o\delta(r) \tag{3}$$

so that the "shell" barrier strength is V_o, and the central delta-function has strength U_o. Both potentials are repulsive, ie., $V_o, U_o > 0$, so there are no bound states - however there will be resonant states inside the shell. In what follows you may find it useful to model the central delta-function by a potential $U(r) = (U_o/\pi a_o^2)\theta(a_o^2 - r^2)$, and then let $a_o \to 0$.

(i) Find the form of the scattering functions $f_l(E)$ for the l-th angular momentum partial waves.

(ii) Now find the form of the $l = 0$ function $f_0(E)$ in the low-momentum limit, where $kR_o \ll 1$; and also the form of the scattering cross-section.