Phys 501: HOMEWORK ASSIGNMENT No (4)

Thursday April 8th 2010

DUE DATE: Monday April 19th 2010.
Assignments handed in late may not receive a full mark.

QUESTION (1): TIME-DEPENDENT PERTURBATION THEORY

Consider a spin $\mathbf{S} = \hbar\boldsymbol{\sigma}$, where $|\boldsymbol{\sigma}| = 1/2$, which for time $t \to -\infty$ is in a state oriented along the $\hat{z}$ axis, and which is subject to a field $\mathbf{B} = \hat{\mathbf{z}}B_o + \hat{\mathbf{x}}b_1 f(t)$, where $|b_1| \ll |B_o|$, and where

$$f(t) = \frac{e^{t/t_o}}{1 + e^{t/t_o}} \tag{1}$$

for all times $-\infty < t < \infty$.

(i) Using first-order time-dependent perturbation theory, find the amplitude $c_-(\infty)$ for the spin to be in the state oriented along $-\hat{z}$ as time $t \to \infty$ (evaluate the Fourier transform by closing the contour in the upper half-plane).

(ii) Suppose that $g\mu_B|B_o|t_o \ll \hbar$; we can then find the exact dynamics of the system, at any time t, by elementary means. What is this dynamics?

QUESTION (3): SPIN TUNNELING

Suppose we have a spin of magnitude S in a combined easy-axis anisotropy field and a transverse term, so that the Hamiltonian is given by

$$\mathcal{H}_o = -K_o\hat{S}_z^2 + E_o\hat{S}_x^2 \tag{2}$$

where $K_o, E_o > 0$

(i) Find the energy levels of the problem when $E_o = 0$

(ii) Suppose now that $E_o > 0$, but $E_o/K_o \ll 1$. Show that if S has an integer value, then pairs of levels which have projection $S_z = \pm M$ are split by the transverse term, with a splitting given by perturbation theory in the parameter E_o/K_o as

$$\Delta_M^S \; = \; A_M^S K_o \left(\frac{E_o}{16K_o}\right)^M \quad ; \quad A_M^S = \frac{8}{[(M-1)!]^2}\frac{(S+M)!}{(S-M)!} \tag{3}$$

and find an approximate expression for the ground-state splitting Δ_S^S when $S \gg 1$.

(iii) The Hamiltonian in (ii) can be diagonalised numerically very easily. Do this for $S = 20$, and plot $ln\Delta_S^S$ as a function E_o/K_o. Compare with the answer in (ii) to see how accurate the perturbative expansion is.