Phys 501: HOMEWORK ASSIGNMENT No (3)

Sunday Feb 15th 2009; revised Saturday 28 March

DUE DATE: Monday March 9th 2009.

Assignments handed in late may not receive a full mark.

QUESTION (1): BOUND STATES

(i) Suppose we have a 'hard well' 2-d centrally potential well of form

$$V(r) \;=\; 0 \quad (r < R); \qquad\qquad V(r) \;=\; \infty \quad (r > R) \tag{1}$$

Find the single-particle eigenstates and eigenenergies for this system.

(ii) Now let's consider a 2-dimensional 'circular' potential well, in which there is a repulsive core, with form

$$V(r) \;=\; V_o\delta(r) + V_o\theta(a_o^2 - r^2) \qquad\quad (V_o < 0) \tag{2}$$

Find the solutions (eigenfunctions and eigenenergies) to this problem - this includes both bound states and extended states.

QUESTION (2): 1-DIMENSIONAL POTENTIALS

(i) Consider a 1-dimensional "double-barrier" potential of form

$$V(x) \;=\; V_o[\delta(x - a_o/2) + \delta(x + a_o/2)] \tag{3}$$

where each δ-function barrier has the same strength.

Find the solutions (eigenfunctions and eigenenergies) to this problem assuming we have a plane wave coming in from the left.

QUESTION (3): TWO-DIMENSIONAL POTENTIAL with FLUX TUBE

(i) Consider the problem of a 2-d 'circular' potential with the form

$$V(r) \;=\; 0 \quad (r > R); \qquad\qquad V(r) \;=\; V_o \quad (r < R) \tag{4}$$

This may either be a repulsive circular barrier (for $V_o > 0$), or an attractive circular well (for $V_o < 0$); you just solved the attractive case above, and the repulsive case is in the notes.

Now suppose we add to the problem another potential, coming from a flux tube of infinitesimal radius, but containing a total flux $\Phi = \alpha\Phi_o$, where $\Phi_o = h/e$ is the flux quantum and e the charge of the particle (if the radius of the flux tube is a_o, then the field has strength $B(r) = (\Phi/\pi a_o^2)\theta(a_o - r)$; we then let $a_o \to 0$).

Solve now the problem for the particle moving simultaneously in the circular potential and the flux tube potential just specified. You should do this separately for the repulsive and attractive cases.