Phys 501: HOMEWORK ASSIGNMENT No (2)

Sunday January 25th 2009

DUE DATE: Friday Feb 6th 2009.
Assignments handed in late may not receive a full mark.

QUESTION (1): RANDOM FORCE PROPAGATOR

(i) Consider a free particle which is acted upon by a force $F(t)$ so that its Lagrangian is

$$L \;=\; \frac{m}{2}\dot{x}^2 - F(t)x \tag{1}$$

Suppose that at time t_1 the system is at x_1, and at time t_2 it is at x_2. Find the classical action for a particle with these boundary conditions, following the classical trajectory of minimum action.

(ii) Now, derive an expression for the quantum propagator $G(2,1)$ for a particle with the same Lagrangian and these same boundary conditions, including the prefactor.

QUESTION (2): THE BIASED 2-LEVEL SYSTEM

(i) Consider the 2-level system with Hamiltonian

$$\mathcal{H} = \epsilon_o \hat{\tau}_z + \Delta_o \hat{\tau}_x \tag{2}$$

Find the eigenfunctions and eigenvalues of this Hamiltonian, and then the time-dependent density matrix, assuming the system starts in the state $|\uparrow\rangle$ at $t = 0$.

(ii) Now write down a path integral expression for the time evolution of the wave-function, assuming the same initial wave-function as above; and rederive the same result for the density matrix as a function of time.

QUESTION (3): PAIR of OSCILLATORS

(i) Consider a pair of coupled oscillators with the Hamiltonian

$$\mathcal{H} \;=\; \frac{1}{2m}(p_1^2 + p_2^2) + \frac{m}{2}(x_1^2 + x_2^2 + 2g x_1 x_2) \tag{3}$$

where $g < 1$. Write this in second quantised form, and then find the eigenenergies of the problem, and show how the Hamiltonian can be written in terms of a set of new boson operators which also satisfy the usual commutation values.

(ii) Now write down a set of coherent states for this new system, and show in a figure what kind of motion this system will show in phase space if it is in a coherent state.