Phys 501: HOMEWORK ASSIGNMENT No (1)

Saturday January 10th 2009

DUE DATE: Friday Jan 16th 2009.
(Please note that assignments handed in late may not receive a full mark.)

QUESTION (1): DENSITY MATRIX FOR SPIN-$1/2$

Imagine that you combine 2 molecular beams, each contaning an equal number of spin-1/2 particles. In one of the beams you know that the spins are all oriented in the $\hat{y}$-direction, in pure states. In the other all you know is that there is an equal statistical mixture of spin-up and spin-down particles.

(i) Find the density matrix of the combined system - give the form for this first in a representation where $\hat{\sigma}_z$ is diagonal, and then where $\hat{\sigma}_y$ is diagonal.

(ii) Now compute the expectation values of $\hat{\sigma}_z$, $\hat{\sigma}_x$, and $\hat{\sigma}_y$, for this system.

QUESTION (2): REDUCED DENSITY MATRIX

Consider a pure state for a pair of spin-1/2 systems, equal to

$$|\psi\rangle \;=\; \frac{1}{\sqrt{3}}[|\uparrow\uparrow\rangle + e^{i\chi}|\downarrow\uparrow\rangle + e^{i\phi}|\downarrow\downarrow\rangle] \tag{1}$$

with 2 different phases χ and ϕ.

(i) Find the total density matrix for the pair of systems, quantizing along the $\hat{z}$-axis.

(ii) Now find the reduced density matrix for the first spin, again quantizing along the $\hat{z}$-axis.

(iii) Now calculate the expectation values of $\hat{\sigma}_z$ and $\hat{\sigma}_x$ for the first spin using this reduced density matrix.