Phys 501: HOMEWORK ASSIGNMENT No (1)

Tuesday January 12th 2010

DUE DATE: Wednesday Jan 20th 2010.

(Please note that assignments handed in late may not receive a full mark.)

QUESTION (1): CLASSICAL MECHANICS

(i) State the principle of least action, and derive Lagrange's equations from this principle, for a system having coordinates $Q \equiv (q_1, q_2, .., q_{3N})$.

(ii) Now consider a pendulum, with a mass M hanging at the end of a weightless rigid rod of length l, and allowed to move in a single plane only, with angular displacement θ from the lowest energy vertical configuration. Write down the action, the Lagrangian, and the Hamiltonian for this system (no need to evaluate the action here, just write it in integral form). Then derive the equations of motion for the system, and give the general solution for these in the regime where $\theta \ll 1$.

(iii) Now construct the phase portrait of this system, as a function of the angular variable. Show both the lines of constant action, and also the 'streamlines' showing the motion of the system as a function of time.

QUESTION (2): DENSITY MATRICES

(i) Consider a 2-spin system of spin-1/2 particles, which has been prepared such that we have (a) probability A that the system is in an incoherent mixture, with equal weighting, of states $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$, and (b) probability $1 - A$ the system is in a superposition with wave-function $\psi = [|\uparrow\uparrow\rangle + e^{i\theta}|\downarrow\downarrow\rangle]/\sqrt{2}$.

Find the total density matrix for this 2-spin system, quantizing along the $\hat{z}$-axis. Then calculate the expectation values of (a) the operator $\hat{\sigma}_1^z \hat{\sigma}_2^z$, and (b) the operator $\hat{\sigma}_1^x \hat{\sigma}_2^x$, for the pair of spins.

(ii) Now suppose we trace over spin 2, to get a reduced density matrix for spin 1. Find this reduced density matrix, and then calculate the expectation values of $\hat{\sigma}_1^z$, and $\hat{\sigma}_1^x$, for the first spin.