Phys 501: HOMEWORK ASSIGNMENT No (2)

Wednesday March 7th 2012

DUE DATE: Monday March 19th 2012.

(Please note that assignments handed in late may not receive a full mark.)

QUESTION (1): SCATTERING OFF 'SHELL' POTENTIALS

(i) Consider a potential of form

$$V(x) \;=\; V_o\delta(x) - U_o\theta(a_o^2 - x^2) \tag{1}$$

where $V_o, U_o > 0$, and $\theta(x)$ is the Heavisde function. Now assume we have a plane wave incident to the left on this potential. Find the scattered solution for $x > a_o$, and thereby determine the T-matrix for this problem, as a function of the wave-vector k of the incoming wave, and of the parameters V_o, U_o, and a_o.

Draw a graph of the way in which the phase shift δ_o depends on ka_o, for different values of V_o/U_o, under circumstances where there is a single bound state in the potential well.

(ii) Now consider a somewhat similar 2-d version of this problem, to a δ-shell potential, given by $V(r) = V_o\delta(|\mathbf{r}| - a_o)$. This problem is actually solved explicitly in the course notes. However we can modify it as follows - let the potential now become

$$V(r) \;=\; V_o\delta(|\mathbf{r}| - a_o) \;+\; U_o\delta(\mathbf{r}) \tag{2}$$

where $U_o > 0$.

Now solve this problem, and find a formula for the phase shifts $\delta_l(k)$ as a function of k and of the strengths U_o and V_o of the 2 parts of the potential.

To understand these results, let's first compare with the result when $U_o = 0$. Show in a graph how both $\delta_o(k)$ and $t_o(k)$ vary as a function of k for a number of different strengths of the shell potential V_o, when $V_o = 0$.

Now do the same for a given value of V_o while varying U_o. From these graphical investigations, see if you can give a qualitative picture of what is going on here.

QUESTION (2): T-MATRIX PROBLEM

Consider a potential whose magnitude as a function of energy ϵ_k can be written as $V(\epsilon_k) = V_o\;\theta(D_o^2 - \epsilon_k^2)$, where $\theta(x)$ is the Heaviside function, in a problem where the electrons themselves are given a density of states $N(\epsilon) = (N_o/2D_o)\;\theta(D_o^2 - \epsilon^2)$ in energy space. This roughly describes impurity scattering in a conductor of bandwidth $2D_o$.

(i) Find the phase shift $\delta_o(z)$ and the T-matrix $T(z)$ as a function of complex energy z for this system. Draw the phase shift as a function of real energy (ie., letting $z \to E$, where E is real).

(ii) Then find an expression for the one-particle Green function $G(z)$ for the same system, and for its imaginary part $A(E)$ on the real axis. Draw a picture showing the pole structure of the Green function on the complex energy sheets.