Phys 501: HOMEWORK ASSIGNMENT No (2)

Friday Feb 11th 2011

DUE DATE: Friday Feb 25th 2011.

(Please note that assignments handed in late may not receive a full mark.)

QUESTION (1): PROPAGATOR IN A STATIC FIELD

(i) The Lagrangian for a particle of charge q and mass m, in a static field, is given by

$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 + q\dot{\mathbf{r}} \cdot \mathbf{A}(\mathbf{r}) - qV(\mathbf{r}) \tag{1}$$

where $\mathbf{A}(\mathbf{r})$ is the magnetic vector potential, and $V(\mathbf{r})$ is the electric potential.

Write down the path integral expression expression for the 1-particle Green function $G(\mathbf{r}_2, \mathbf{r}_1; \tau)$. Then, by considering an infinitesimal change in the final time t_2, show that the Schrodinger equation for the particle has the form

$$\left[\frac{1}{2m}[(-i\hbar\nabla - q\mathbf{A}) \cdot (-i\hbar\nabla - q\mathbf{A})] + qV(\mathbf{r})\right] \psi(\mathbf{r}, t) = -i\hbar\partial_t\psi(\mathbf{r}, t) \tag{2}$$

(ii) The Lagrangian given above for a charged particle in a field is a quadratic function of its arguments, provided the vector potential is also a quadratic function of $\mathbf{r}$. One may then derive an exact expression for the propagator in terms of the classical action, and the fluctuation prefactor, evaluated to quadratic order in fluctuations, in the form

$$G(\mathbf{r}_2, \mathbf{r}_1; \tau) = A(\tau)e^{iS_c(2,1)/\hbar} \tag{3}$$

Show that if the magnetic field is a constant along the $\hat{z}$-axis, so that $\nabla \times \mathbf{A}(\mathbf{r}) = \hat{z}B_o$, then by choosing the appropriate gauge, the terms in the propagator take the form

$$A(\tau) = (m/2\pi\hbar\tau)^{3/2}\frac{\omega_c\tau}{2\sin(\omega_c\tau/2)} \tag{4}$$

$$S(2,1) = \frac{m\omega_c^2}{4}\cot(\omega_c\tau/2)\left[|\mathbf{r}_2 - \mathbf{r}_1|^2 - \hat{z} \cdot (\mathbf{r}_2 \times \mathbf{r}_1)\right] \tag{5}$$

where $\omega_c = qB_o/m$.

QUESTION (2): SCATTERING PROBLEM

The famous Yukawa interaction (mediated by mesons) can be approximated by ignoring retardation in the interaction, to treat it as a static potential. In this case we have a d-dimensional potential

$$V(r) = \frac{V_o}{r^{(d-1)/2}}e^{-\kappa r} \tag{6}$$

(i) Find the scattering function $f_k(\Omega)$ for the system in the Born approximation, for $d = 1, 2, 3$, where Ω is the scattering angle, and $k = |\mathbf{k}|$, where $\mathbf{k}$ is the incoming momentum. Also find an expression for the phase shifts in Born approximation in $d = 1, 2, 3$ dimensions, and write down the partial wave expansion of the scattered wave. Finally, find the cross-section for the scattering in Born approximation for $d = 1, 2, 3$

(ii) Now consider the problem in $d = 1$. Find an expression for the T-matrix of the system in this case. Under what circumstances can you accurately sum the infinite series here, and what answer do you then get?