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PHYS 403: HOMEWORK ASSIGNMENT No. 4: FERMIONS and PHASE TRANSITIONS

(Mar. 31st, 2023)

HOMEWORK DUE: FRI, April 14, 2023 (revised deadline)

To be uploaded by 11.59 pm, Friday, April 14th - Late Homework will not be accepted

QUESTION (1) DEGENERATE FERMIONS:

1(a): You are given a metal which has a density of conducting electrons of $10^{29}\ m^{-3}$. Treating these electrons as thoiugh they were a non-interacting gas of fermions, find the $T \rightarrow 0$ values for (i) the Fermi energy, in eV; (ii) the "Fermi wavelength" of electrons at the Fermi surface, and (iii) the density of states at the Fermi energy.

1(b): A popular model for a metal (because it is so simple) is to pick a density of states:

$$g(E) = (N_o/D_o)\,[\theta(E + D_o) - \theta(E - D_o)]$$

Here the energy $2D_o$ is called the "bandwidth" of the metal, and N_o is the number of conduction electrons per unit volume; we also assume that the $T = 0$ chemical potential is at energy $\mu = 0$.

Derive, for this system, the electronic specific heat $C_V(T)$ and the temperature dependence of the chemical potential $\mu(T)$, for low temperatures (ie., for $k_BT \ll D_o$), up to terms $\propto T^2$.

1(c): A popular model for a semiconductor has a density of states (with A_o, B_o both constants):

$$g(E) \;\;=\;\; g_o\,[A_o\theta(E - \Delta_o)\sqrt{E - \Delta_o} \;+\; B_o\theta(-E - \Delta_o)\sqrt{-E - \Delta_o}\,]$$

We assume that at $T = 0$, all states in the lower "valence band" are full, whereas all states in the upper "conduction band" are empty. Show that (i) if $A_o = B_o$, then $\mu(T) = 0$ for any value of T: whereas if $A_o > B_o$, $\mu(T) < 0$ for $T > 0$.

QUESTION (2) QUANTUM-CLASSICAL CROSSOVER: A very simple model of transitions through a potential barrier, out of a potential well, makes the following crude approximations. We assume (i) that the energy levels are equally-spaced inside the potential well (with energies $\epsilon_n = -n\epsilon_o$), so that the lowest-energy state is at an energy $E_N = -N\epsilon_o$, and the highest energy state has energy $E_o = 0$); and (ii) we assume that we can approximate the shape of the barrier as $V(x) = E_o - \alpha x^2$, with $E_o = 0$ as measured from the top of the barrier, in all of the regions that are important for tunneling, right down to the bottom of the barrier (and of the potential well). You may find it helpful to draw a sketch of the potential.

2(a): For a particle in the n-th level in the well, at energy $-n\epsilon_o$, find the WKB tunneling probability $\Gamma_Q^{(n)}$ though the barrier at this energy (assume the particle has mass M), using the form given above for the barrier potential $V(x)$.

2(b): Now, let us assume that the total transition rate $\Gamma(T)$ out of the potential well is given by

$$\Gamma(T) \;=\; \sum_{n=0}^{N} \Gamma_Q^{(n)}\, \exp[\beta\epsilon_n]$$

Here $\beta = 1/k_BT$, and we sum over all the levels from the lowest energy state at energy $E_N = -N\epsilon_o$, up to the highest energy state at the top of the barrier, at energy $E_o = 0$ (and the sign in positive in the exponent because we are defining our energies ϵ_n with negative sign).

Show that at low T the transition is dominated by the transitions from the lowest state at energy $E_N = -n\epsilon_o$, whereas at high T, it is dominated by transitions from the state at the top of the barrier, at energy $E_o = 0$. Show also that there is an intermediate ‘crossover’ temperature T_c at which the transition rate from each of the levels is roughly the same; and find T_c as a function of the parameters α and k_B.

END of 4TH HOMEWORK ASSIGNMENT