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PHYS 403: HOMEWORK ASSIGNMENT No. 3: PHOTONS and FERMIONS

(Mar. 27th, 2022)

HOMEWORK DUE: FRIDAY, April 8th, 2022

To be uploaded by 11.59 pm, Friday April 8th - Late Homework will not be accepted

QUESTION (1) The UNIVERSE: Suppose we think of the universe as a closed homogeneous volume, currently with a radius $R_U \sim 1.5 \times 10^{10}$ light yrs $\sim 1.5 \times 10^{23}$ km. The photon gas in it is at an equilibrium temperature $T_U \sim 3$ K.

1(a) Derive an expression for the total number N of thermal photons in the universe, and find this number, using the information just given.

1(b) What is the expression for the total energy U of these thermal photons? Find this energy, in MKS units.

1(c) The universe is expanding; suppose we have roughly $R_U(t) \propto t$. It is usually assumed that the photon gas expands adiabatically with the universe. Why can we assume this? Now, show that we have for the photon gas that

$$T \left(\frac{\partial S}{\partial T} \right) \propto VT^3 \tag{0.1}$$

where V is the volume of the universe; and hence that the entropy $S \propto VT^3$ is a constant.

Given this result, what was the temperature of the thermal photons when the universe was 15 million yrs old? And what will it be when the age of the universe is 120 trillion (1.2×10^{14}) yrs old?

QUESTION (2) FERMI GASES:

2(a): Consider a white dwarf of radius 20,000 km. Let's suppose that it is composed entirely of protons and electrons, is charge neutral, and has a mass of 2×10^{30} kg (ie., 1 solar mass). What is the Fermi energy of the electrons in units of (i) Joules (ii) electron Volts, and (iii) degrees Kelvin? Assume a uniform density for the system (which is of course not the case), and a non-relativistic energy dispersion for the electrons.

2(b) What is the Fermi energy of the protons in this white dwarf, in eV units, if the proton is 1,836 times more massive than the electron?

2(c) For a proton and an electron to transform into a neutron, we require the reaction $p + e^- + K.E. \rightarrow n + \nu_e$, where ν_e is the electron neutrino. The minimum extra kinetic energy K.E. required - which will be an electronic kinetic energy - is the mass difference $(m_n - m_p)c^2 = 0.78$ MeV.

Show that this implies a minimum possible mass density for the neutron star, and find what this is in MKS units. Assume non-relativistic energy dispersion laws for the particles.

END of 4th HOMEWORK ASSIGNMENT