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PHYS 403: HOMEWORK ASSIGNMENT No. 3: BOSONS

(Mar. 14th, 2022)

HOMEWORK DUE: THU, March 30th, 2023 (revised deadline)

To be uploaded by 11.59 pm, Thursday March 30th - Late Homework will not be accepted

QUESTION (1) PHOTON BEAMS: We decide to make a very crude monochromatic photon beam simply by collimating and filtering light from a black body. Suppose we have a spherical black body of diameter 1m at temperature T, and we collimate the light from this body through a circular hole hole of radius 1 cm, at a distance of 10m from the centre of the black body. We use a filter of wavelength bandwidth $\Delta\lambda = 0.01\mathring{A}$, centred on a wavelength $\lambda = 5890\mathring{A}$ (ie., one of the 2 main Na lines).

1(a) Suppose we want to have a power of 10^{-4} W in this beam. What temperature must the black body be at to get this?

1(b) What would be the difference between this light and the light from a laser of the same frequency and the same linewidth be? How could you tell the difference in an experiment?

QUESTION (2) PHOTONS: It is also interesting to see how the behaviour of photons differs from that of massive bosons - in particular, superfluidity and Bose condensation are impossible for photons.

2(a): Explain under what circumstances, and why, it is that $\mu = 0$ for photons. Where do we expect to see photons for which $\mu \neq 0$, and why?

Now, consider a photon gas in n dimensions, where n is a positive integer. Derive expressions for (i) the density of states $g(\epsilon)$, and (ii) the energy density $u(T)$, for a photon gas in n dimensions; and show that $u(T) \propto T^{n+1}$.

2(b) Suppose now that the density of states $g(\epsilon) = g_o\,\epsilon$ per unit area A for a 2-dimensional system of photons (something easily arranged by having it in a confined geometry). Find expressions for (i) the grand canonical potential $\Xi(T)$ per unit area, (ii) the total energy $U(T)$, and (iii) the pressure $p(T)$, for this photon gas. What is the relationship between p, U, and A?

QUESTION (3) INFLUENCE of DENSITY of STATES: The 2 key factors which decide the behaviour of a set of bosons or fermions are (i) the value of the chemical potential $\mu(T)$, and (ii) the form of the density of states $g(E)$. Let's look at how these influence the results for a set of non-interacting particles.

3(a) Suppose we have a set of N bosons, with N conserved, and $g(\epsilon) = g_o\epsilon^{1/2}$. Write down an expression for the number density $\rho = N/V$ of particles for this system, and explain why the system will show Bose condensation. Find the temperature T_c at which Bose condensation occurs, and find the temperature dependence of the condensate density $\rho_s(T)$ as a function of T for $T < T_c$. You can use the result that

$$\int_0^\infty dx \frac{x^{1/2}}{e^x - 1} \;=\; A_o \;=\; 1.306\pi^{1/2}$$

3(b) Suppose now we have a density of states given by: $g(\epsilon) = \frac{g_o}{E_1 - E_2}\left[\theta(\epsilon - E_1) - \theta(\epsilon - E_2)\right]$, where $\theta(x)$ is the step function. Again, write down an expression for ρ for this system, assuming a set of non-interacting bosons.

Suppose that instead we were dealing with fermions, and that $E_2 > \mu > E_1$. What then would be the relationship between $\rho(T)$, μ, and g_o? Suppose now that for some reason we have $\mu(T)$ pinned so that $\mu - E_1 = (E_2 - E_1)/2$; what then is thre zero temperature value $\rho(T = 0)$ in terms of g_o? And what then is the total energy $U(T)$ of the system in an expansion up to $\sim O(T^2)$, assuming that $k_B T \ll (\mu - E_1)$?

3(c) Finally put $E_1 = 0$ and $E_2 = \infty$ for the density of states just given, and assume again a set of non-interacting bosons. Show, from your expression for ρ in this case, that the system will NOT show Bose condensation, by rewriting the integral over energy in the form of an infinite sum (expand the Bose function as a power series).

END of 3RD HOMEWORK ASSIGNMENT