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PHYS 403: HOMEWORK ASSIGNMENT No. 1: PROBABILITIES, THERMODYNAMICS, and MICROSTATES

(Jan. 22nd, 2023)

HOMEWORK DUE: MONDAY, Feb 6th, 2023

To be uploaded by 11.59 pm, Monday Feb 6th - Late Homework will not be accepted

QUESTION (1) DISCRETE PROBABILITIES: Consider the following problems:

1(a): Suppose I throw 8 equally weighted 6-sided dice. What is the probability that I will get the number 1 showing up twice, 3 showing up once, 4 showing up twice, 5 showing up twice, and 6 showing up once?

1(b): Suppose you are dealt a hand of 5 cards (the "first draw") from a randomized pack of 52 cards (the usual pack here, with aces, kings, queens, jacks, and numbers from two to ten).

What is the probability that you will get "3 of a kind", eg., three aces, or three 8's, etc., along with 2 other cards which are different from these?

1(c): Suppose you do get 3 of a kind in this first draw; but now you are allowed to throw away the 2 other cards, and get two others in their place (the "second draw"). What is the probability that (a) these 2 other cards will be a "pair" (eg., two kings, or two 4's); and (b) alternatively, what is the probability that one of these 2 other cards will have the same value as the three of a kind you already have (eg., if you already have 3 aces, what is the probability that one of the two extra cards you draw will be the 4th ace)?

QUESTION (2) THERMODYNAMICS for a MAGNETIC SYSTEM: Suppose we have a magnetic system whose equation of state is $M(T, B) = CB/T$, where M is the magnetization, B the magnetic field, T the temperature, and C is a constant. The energy of this system is just $U = -MB$, and if the field B is changed, the work done by the system is $dW = MdB$.

2(a): Show that the heat dQ given to the system under simultaneous changes dB and dM is $dQ = -BdM$.

2(b): From this find the change dS and the form for the entropy $S(M)$ for the system.

QUESTION (3) N SPIN-1/2 SYSTEMS: Consider a set of N non-interacting spin-1/2 systems in a magnetic field, such that the energies of each individual spin are E_1 and E_2.

3(a): Find the partition function for this system, and, at temperature T, find the average energy $U(T)$ for the total system. From this derive also the specific heat $C_V(T)$.

3(b): Find expressions for $U(T)$ and $C_V(T)$ when $kT \gg |E_1 - E_2|$. You should find the $T = \infty$ result, and also the first correction to this result, for finite (but very large) T.

END of 1ST HOMEWORK ASSIGNMENT