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PHYS 403: HOMEWORK ASSIGNMENT No. 1: PROBABILITIES, THERMODYNAMICS, and MICROSTATES

(Jan. 22nd, 2022)

HOMEWORK DUE: SUNDAY, Feb 6th, 2022

To be uploaded by 11.59 pm, Sunday Feb 6th - Late Homework will not be accepted

QUESTION (1) MULTINOMIAL DISTRIBUTION: This question is to help you with calculating probabilities for a finite set of discrete outcomes.

1(a) Suppose I throw 10 equally weighted 6-sided dice. What is the probability that I will get 2 showing up twice, 3 showing up once, 4 showing up three times, 5 showing up twice, and 6 showing up twice?

1(b) Suppose I have 4 political parties in a country, and in an election the parties A,B,C,D receive fractions 0.4, 0.3, 0.2, and 0.1 of the total vote respectively. I now decide to take a rather limited poll of 10 people. What is the probability that in this poll, I will get 4 people who voted for A, 3 people who voted B, 2 people who voted C, and 1 person who voted D?

QUESTION (2) THERMODYNAMICS for an IDEAL GAS: Consider a container of 2 moles of an ideal monatomic gas, whose volume is initially V_o, and whose pressure is initially p_o. We then expand the gas to a volume 10 V_o.

2(a) Suppose first that the expansion is done isothermally, with the gas staying at temperature T_o throughout. Find the work done by the gas, and the heat absorbed by the gas, during the course of the expansion. What is the total change in internal energy of the gas?

2(b) Now suppose that the expansion is done at constant pressure p_o. Find again the work done by the gas, and the heat absorbed by the gas, during the course of this expansion. What is the total change now in internal energy of the gas?

QUESTION (3) N-SPIN SYSTEM: Consider a set of N non-interacting spin-S systems in a magnetic field, such that the energy ϵ_m of each individual spin is $\epsilon_m = m\Delta_o$, where Δ_o is a constant, and $m = S, S-1, \cdots, -S$ can take one of $2S+1$ values.

3(a) What are the allowed energies U_j of the total system of N spins?

3 (b) Find $W(U_j)$, the number of available microstates to the N-spin system having energy U_j, where U_j is one of the allowed energies you found in 3(a).

3(c) Using the relation that the entropy $S = k_B \ln W$, And assuming that $N \gg 1$, find an approximate expression for the entropy S as a function of the energy U of the system.

END of 1ST HOMEWORK ASSIGNMENT