PHYS 403: REVISION QUESTIONS (short) 2022

NOTES: The exam is divided into 6 short questions (in section A) and 4 long questions (in section B). You must answer a total of **THREE** short questions from section A, and **TWO** long questions from section B. You can choose which questions you decide to answer. Note that extra marks will not be given for answering more than 4 questions in section A or 2 questions in section B; if you do, we will simply choose those questions which give you the highest mark.

Here is a set of short questions. You should take ROUGHLY 20 mins to do any one of them (some of them may take a little longer). I will send out answers to them all in a few days.

SECTION A: SHORT QUESTIONS

QUESTION A.1: QUANTUM GASES

(i): Why does the diameter of a white dwarf decrease when its mass increases?

(ii) Why does the chemical potential of a gas (Bose, Fermi, or classical) never increase (and almost always decreases) as one raises the temperature?

QUESTION A.2: 2-LEVEL SYSTEMS

(i): Consider a set of N non-interacting 2-level systems (TLS), with level energies E_1 and E_2 for each of the TLS. At temperature T, what is the average energy $U(T)$ for the total system? Derive also the specific heat $C_V(T)$.

(ii) Find expressions for $U(T)$ and $C_V(T)$ when $kT \gg |E_1 - E_2|$. You should find the $T = \infty$ result, and also the first correction to this result, for finite (but very large) T.

QUESTION A.3: FERMI DISTRIBUTION

(i): The grand canonical partition function for a single fermion state of energy ϵ is $z(\epsilon) = \sum_n \exp[n\beta(\mu - \epsilon)] = 1 + \exp[\beta(\mu - \epsilon)]$. Show that the mean occupation number for this state is just the Fermi function, ie., that $\langle n \rangle \rightarrow f(\epsilon - \mu) \equiv \{1 + \exp[\beta(\epsilon - \mu)]\}^{-1}$, which we also write as $f(x) = [1 + e^{\beta x}]^{-1}$, where $x = (\epsilon - \mu)$.

(ii) Then show that the probability of finding n particles in this state is

$$p(n) \; = \; \frac{[1 - f(-x)]^n}{[f(-x)]^{n-1}}$$

QUESTION A.4: INTERATOMIC POTENTIAL

(i): Consider the 1-dimensional potential

$$V(x) \; = \; V_o \left[\left(\frac{a_o}{x} \right)^{12} - 2 \left(\frac{a_o}{x} \right)^6 \right]$$

Find the value of x for which $V(x)$ is a minimum, and find the "curvature" d^2V/dx^2 at this point. What is the frequency of small oscillations of a particle of mass M about the minimum in this potential?

(ii) Draw a picture of the potential $V(x)$, and explain briefly how it can be used to model interatomic interactions. For such interaction, what do you think are typical values for V_o and a_o?

QUESTION A.5: ARGON in ATMOSPHERE

(i): Roughly 1 percent of the volume of the earth's atmosphere is composed of ^{40}Ar. Suppose you are in a bedroom with a volume of 60 m^3. Roughly how many ^{40}Ar atoms are in the room, and what is their total mass?

(ii) In MKS units, roughly what is the total thermal energy associated with the ^{40}Ar atomic motion?

QUESTION A.6 BLACK BOXES:

(i) Suppose I surround the sun (considered to be a black body at a temperature T_s) with a perfectly black shield. Assuming the shield is indestructible, it will come to an equilibrium temperature T_B, governed by the radiative energy flow from the sun and the energy radiated by the shield. Assuming that the universe outside the shield can be treated as a reservoir at temperature $T = 0$, show that the equilibrium temperature of the shield is $T_B = (T_s/2)^{1/4}$. Assume the shield is close to the sun's surface, so that the surface area of the shield is the same as that of the sun.

(ii) Now suppose instead I replace the single shield with 2 concentric shields. What is the temperature T_1 of the first (inner) shield, and T_2 for the second outer shield?

(ii) Finally let us generalize the argument to a set of N concentric black shields. What is the temperature T_n of the n-th shield, for $1 \geq n \geq N$?

QUESTION A.7 FREE ENERGY: For a gas of particles, the infinitesimal changes dS in the entropy and dV in the volume of the gas container result in a change dU in the energy, given by $dU = TdS - pdV$, where T is the temperature and p the pressure in the gas.

(i) The free energy of the system is $F = U - TS$. Find an expression for the infinitesimal dF, and show that the pressure is then given by the partial derivative $p = -(\partial F/\partial V)|_T$, where T is held constant.

(ii) Suppose we are allowed to add particles to the gas as well, so that $dU = TdS - pdV + \mu dN$, where μ is the chemical potential of the gas particles and N is their total number in the container. Assuming again that $F = U - TS$, find an expression for μ in terms of a partial derivative of F; make sure to specify what is held constant.

QUESTION A.8 QUALITATIVE PHYSICS: Be quantitative if you can - but the answers also require qualitative physical understanding.

(i): Why does the entropy of a pair of systems usually increase when one combines them physically into a single system? When does it not increase? If one then takes a single system, and separates it into two systems, the entropy also increases - why is this?

(iI): Suppose I have two sets of spin-1/2 systems, each containing N spins in an applied magnetic field which splits each spin level by an amount $2\Delta_o$. We assume that they are in every way identical except that one of the N-spin systems is at temperature $T = 0$, while the other is at temperature $T = \infty$.

Now, I combine the 2 sets of spins. What is the final energy of the combined system? And if I do the combination in an irreversible way, what do you think is the final temperature?

QUESTION A.9: RADIATION PRESSURE The radiation pressure p from photons is equal to $p = 4J/3c$, where J is the radiation flux. A star like the sun emits black-body radiation with flux $J = \sigma T^4$ per unit area of its surface, where temperature T is measured in Kelvin units; here $\sigma = 5.67 \times 10^{-8} W m^{-2} K^{-4}$, and the sun's surface temperature is $6,000\ K$. The radius of the sun is $R_S \sim 0.7 \times 10^6$ km.

(i): Consider the forces on an electron at the sun's surface. If the cross-section for photon-electron scattering is $\sim 6.6 \times 10^{-29}\ m^2$, and the electron mass is $\sim 9 \times 10^{-31}$ kg, then how do the gravitational and radiation forces on the electron at the sun's surface compare (assume here that all the photon energy is taken up by the electron)? You can assume that the solar mass is 2×10^{30} kg, and that the gravitational constant $G = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$.

(ii) How do the radiation force and gravitational force on the electron behave as a function of the distance r from the sun (for $r > R_o$)?. What then is the equation of motion for $r(t)$, and what is its solution as a function of time, if the electron starts at a distance $r_o = r(t=0)$ from the sun?

QUESTION A.10 ENERGY FLUCTUATIONS: We again start from the canonical partition function $\mathcal{Z}$ for a system $\mathcal{S}$.

(i) Show that the "mean energy squared" of the system $\mathcal{S}$ is given by $\langle E^2 \rangle = \mathcal{Z}^{-1}(\partial^2 \mathcal{Z}/\partial \beta^2)$.

(ii) Using this result work out an expression for the mean squared energy fluctuation in the energy of the system, written as $\langle \Delta E^2 \rangle = (\langle E^2 \rangle - \langle E \rangle^2)$; and then show that it can be written as

$$\langle \Delta E^2 \rangle \quad = \quad -\frac{\partial \langle E \rangle}{\partial \beta}$$

END of SHORT QUESTIONS