PHYS 350: HOMEWORK ASSIGNMENT No. 6

(Nov. 26th, 2004)

HOMEWORK DUE: FRIDAY, Dec. 3RD 2004

To be handed in during class- Late Homework will not be accepted

Question (1)

A space station is shaped like a giant bicycle wheel, with negligible mass at the centre, and a mass of 10,000 tons distributed evenly over the entire rim. To good approximation the rim can be assumed to be concentrated entirely at a distance of 100 m from the centre. Assume 1 ton $= 10^3$ kg.

(i) Calculate the moments of inertia, in MKS units, of the space station about its principal axes (ignoring the small contributions from the centre, and assuming the mass concentrated at the rim).

(ii) It is decided to spin the space station up to an angular velocity $\mathbf{\Omega}$, along the axis of symmetry of the wheel, such that the occupants living in the outer rim will feel an outward acceleration equal to the gravitational acceleration on the earth (ie., $10ms^{-2}$). What must Ω be?

If this 'spin-up' of the wheel is done by expelling gas at very high velocity, $10kms^{-2}$, from jets situated on the rim and aligned along the rim tangent. What mass of gas will be required to do this (ignore the slight change in moment of inertia of the wheel caused by loss of gas. Note that the gas is always expelled at $10kms^{-2}$ *relative to the moving rim.*

Question (2)

Consider a long cylinder with moments of inertia (in MKS units) given by $I_3 = 0.05$ (the moment of inertia about this axis), and $I_1 = I_2 = 1$. Suppose the angle between the angular momentum $\mathbf{L}$ and the cylinder axis $\hat{x}_3$ is 45^o, and that in MKS units, $L = |\mathbf{L}| = 50$.

Find the components of the angular velocity. What will be the the motion of the cylinder, as viewed by an *external observer*, who is in an inertial frame which is stationary with respect to the cylinder centre of mass?

Question (3)

In a frame of reference *fixed inside a rigid body*, the equation of motion for the angular momentum $\mathbf{L}(t)$, in terms of the angular velocity $\mathbf{\Omega}(t)$ and the the applied torque $\mathbf{N}(t)$, is given by

$$\frac{d}{dt}\mathbf{L}(t) \; + \; (\mathbf{\Omega}(t) \times \mathbf{L}(t)) \;\; = \;\; \mathbf{N}(t) \tag{0.1}$$

(i) Write this equation out explicitly for each component of $\mathbf{L}$. Now, explain why it is that even when $\mathbf{N} = 0$, nevertheless $\mathbf{L}(t)$ is still not necessarily constant in time.

(ii) Suppose that the body is initially rotating with $\mathbf{\Omega}(t = 0)$ very close to the principal axis $\hat{x}_3$ of the system. We now assume a solution for the subsequent motion of the body of the form:

$$\begin{aligned} \Omega_3(t) &= \Omega_o + \eta_3(t) \\ \Omega_2(t) &= \eta_2(t) \\ \Omega_1(t) &= \eta_3(t) \end{aligned} \tag{0.2}$$

where the $\eta_j(t) \ll 1$. We can then substitute this assumed solution into Euler's equations, and expand in the small quantities $\{\eta_j(t)\}$. By linearizing in these small quantities, show that $\mathbf{\Omega}(t)$ oscillates in an ellipse around $\hat{x}_3$, with a frequency ω_o given by

$$\omega_o^2 \;\; = \;\; \Omega_o^2 \frac{(I_3 - I_1)(I_3 - I_2)}{I_1 I_2} \tag{0.3}$$

(iii) Using this answer for ω_o, or by some other means, explain why this oscillatory elliptical motion of $\mathbf{\Omega}(t)$ is actually unstable if $I_1 > I_3 > I_2$.