PHYS 350: HOMEWORK ASSIGNMENT No. 5

(Nov. 12th, 2004)

HOMEWORK DUE: TUESDAY, NOV. 23RD 2004

To be handed in during class- Late Homework will not be accepted

Question (1)

A bicycle with a rigid frame of mass 5kg, and wheels of radius 0.5m and mass 1kg each, is moving at $20ms^{-1}$ along the road, when hits a stationary car head on. After the collision the velocity of the car along the road is $1ms^{-1}$. Neglecting all friction, neglecting the mass and moment of inertia of the car wheels, and assuming all the mass of the bicycle wheels is concentrated at their rims, find out what is the mass of the car.

Question (2)

Suppose we have an extremely long hollow cylindrical pipe of internal radius R. Inside this pipe is a smaller hollow cylindrical tube, of length L, lying coaxial with the pipe, and having internal and external radii of a_1 and a_2 respectively. The tube rolls inside the pipe, without slipping but with no friction. We will also assume that the tube is made from some material having density ρ.

(i) Draw a diagram of the situation just described.

(ii) Find the moment of inertia $I^{\|}$ of the hollow tube along its axis of symmetry, in terms of L, a_1, a_2, and ρ.

(iii) Find the Lagrangian governing the motion of the tube, in terms of $I^{\|}$, its mass M, the gravitational acceleration constant g, and θ, where θ is the angular displacement of the point of contact between the pipe and the tube, measured from a vertical line dropped from the centre of the big pipe (you should show this angle in your diagram in (i)).

(iv) Show that for $\theta \ll 1$ we can write the Lagrangian of the moving tube in the form

$$L \;=\; 1/2[A\dot{\theta}^2 - B\theta^2] \tag{0.1}$$

and so deduce the frequency Ω_o of small oscillations of the tube rolling inside the pipe, in terms of a_1, a_2, g, ρ, and R.