PHYS 350: HOMEWORK ASSIGNMENT No. 4

(Oct. 16th, 2004)

HOMEWORK DUE: TUESDAY, OCT. 26TH 2004

To be handed in during class- Late Homework will not be accepted

Question (1)

Two masses, each of mass M, vibrate because each one is attached to a spring, with spring constants k_1 and k_2 respectively. We also couple them with a very weak spring having spring constant $\kappa \ll k_1, k_2$.

(i) Write down the Lagrangian and equations of motion for this system.

(ii) What are the eigenfrequencies of the system? Imagine we now very the ratio k_1/k_2 over a range from much less than one to much greater than one. Plot the way in which the 2 eigenvalues vary as a function of this ratio.

(iii) Now suppose that $k_1 = k_2$. What are the eigenfunctions of this system?

Question (2)

A spacecraft is approaching the sun, and at time $t = 0$ it is at a distance of $R = 1.5 \times 10^9$ km from the sun (just outside the orbit of Saturn), and it is moving at a velocity $v = 100$ km/sec, travelling in a direction pointing at an angle $\theta = 2^o$ of arc away from the sun. The spacecraft has a mass $m_o = 10^5$ kg; at $t = 0$ an astronaut of mass $m = 100$ kg pushes off from the spacecraft, in the same direction as its motion, at a velocity of 1 m/sec relative to the spacecraft.

(i) Derive an equation for the distance r_o of closest approach for a body coming in to the sun, having an effective potential which depends on the quantities R, v, θ, m_o defined above and also the mass M of the sun (ignore here the mass of the astronaut). No numerical values are asked for here, just an equation for r_o.

(ii) Find the distances of closest approach ("perihelion") of (a) the spacecraft, and (b) the astronaut, to the sun? What are their velocities at these points of closest approach?

(ii) Suppose that on arriving at the distance of closest approach, the spacecraft wishes to go into a circular orbit at this distance from the sun. By how much must much it reduce its velocity to do this? Once it has settled into this orbit, what is the period of the orbit (the time required for a single revolution around the sun)?

NB: In doing these questions you may ignore the effect of the planets, and also ignore the very small recoil of the spacecraft when the astronaut launches from it. In doing (ii) and (iii) you can take the mass of the sun to be 2×10^{30} kg, and the gravitation constant to be $G = 6.7 \times 10^{-11}\ m^3/kg\ s^2$ (more accurate values, if you are interested, are 1.9891×10^{30} kg and $G = 6.67259 \times 10^{-11}\ m^3/kg\ s^2$.