PHYS 350: HOMEWORK ASSIGNMENT No. 3

(Sept. 30th, 2004)

HOMEWORK DUE: TUESDAY, Oct. 12TH 2004

To be handed in during class- Late Homework will not be accepted

Question (1): More on Fourier Transforms

In the usual way we define the Fourier transform of a function $x(t)$ by the formula

$$x(t) \;=\; \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} x(\omega) e^{i\omega t} \tag{0.1}$$

and the inverse transform by

$$x(\omega) \;=\; \int_{-\infty}^{\infty} dt\; x(t) e^{-i\omega t} \tag{0.2}$$

1(a): Consider the "rectangular" function given by $f(t) = [\theta(t+\epsilon) \;-\; \theta(t-\epsilon)]/2\epsilon$, and note that this function encloses unit area, ie., $\int dt f(t) = 1$ (here $\theta(t)$ is the "step-function", or "Heaviside function").

Find the Fourier transform of this function, and draw a graph of it as a function of ω. If we take the limit $\epsilon \to 0$, what are the limiting forms for $f(t)$ and $f(\omega)$? What happens to these 2 functions if instead we take the limit $\epsilon \to \infty$?

1(b): Find the Fourier transform of $x(t) = \theta(t) e^{-\Gamma t} \sin(\Omega_o t)$

1(c): The Green function $G(t-t')$ of the differential operator $\hat{\mathcal{L}} = [A(d^6/dt^6) + B(d^4/dt^4) + C(d^2/dt^2) + D]$ obeys the equation $\hat{\mathcal{L}} G(t-t') = \delta(t-t')$.

(i) What is the simple form for the Fourier transform $G(\omega)$ of $G(t-t')$?

(ii) If a function $x(t)$ obeys the equation $x(t) = \hat{\mathcal{L}} x(t) = f(t)$, then what is the result for the Fourier transform $x(\omega)$ of $x(t)$, in terms of the Fourier transform $f(\omega)$ of $f(t)$?

(iii) Using the last result- if the function $f(t) = \cos(\omega_o t)$, then what is the result for $x(t)$?

Question (2): Coupled Oscillators

Consider a system of 2 rigid pendulums, each with length L, hanging from a horizontal bar, having masses m_1, m_2 at their respective ends, and making small oscillations in a gravitational field. These masses are also coupled to each other by a massless spring, having spring constant k. There is no damping.

2(a): Write down the Lagrangian for this system, and give the equations of motion for the 2 coupled pendulums (you will find it convenient to use the *angular displacement* of the pendulums as coordinates).

2(b): Now, by Fourier transforming these equations of motion, find the 2 "eigenfrequencies" ω_+, ω_- of the motion (ie., the 2 characteristic oscillation frequencies).

2(c): Assume the following boundary conditions at $t = 0$: that the initial state of the system was such that the first pendulum was displaced by an angle of $\theta_1(t=0) = 0.1$ from the vertical, whereas the 2nd pendulum was not displaced at all from the vertical; and both had initial velocity equal to zero. Find the solution for the subsequent motion of the 2 pendulums.

2(d): Now, start again from the equations of motion you derived in 2(a), and this time derive the same results for the motion of the 2 pendulums, by Laplace transforming the equations of motion, and using the same boundary conditions as in 2(c).