PHYS 350: HOMEWORK ASSIGNMENT No. 2

(Sept. 21st, 2004)

HOMEWORK DUE: TUESDAY, SEPT. 28TH 2004

To be handed in during class- Late Homework will not be accepted

Question (1)

We define the Fourier transform of a function $x(t)$ as in the course notes (eqtn (26)) by the formula

$$x(t) \;=\; \int_{\infty}^{\infty} \frac{d\omega}{2\pi} x(\omega) e^{i\omega t} \tag{0.1}$$

and the inverse transform by

$$x(\omega) \;=\; \int_{\infty}^{\infty} dt \; x(t) e^{-i\omega t} \tag{0.2}$$

1(a): Suppose we have a function $x(t)$ which is the *convolution* of 2 other functions $K(t)$ and $f(t)$; this means that

$$x(t) \;=\; \int_{\infty}^{\infty} K(t-t')f(t') \tag{0.3}$$

Now show that the Fourier transform of this convolution is a product; ie., show that:

$$x(\omega) \;=\; K(\omega)f(\omega) \tag{0.4}$$

1(b): Derive the Fourier transforms $x(\omega)$ of the following functions

(i) $x(t) \;=\; A\delta(t-t_1) \;+\; B\delta(t-t_2)$

(ii) $x(t) \;=\; \theta(t-t_1) - \theta(t-t_2)$ (assume that $t_2 > t_1$)

(iii) $x(t) \;=\; A\cos[\omega(t-t_1)] + B\cos[\omega(t-t_2)]$

(iv) $x(t) \;=\; exp[-(t-t_o)]^2$

In these questions, assume that $t_2 > t_1$. The θ-function is just the usual Heaviside step function (ie., $\theta(t-t') = 0$ for $t < t'$, and $\theta(t-t') = 1$ for $t > t'$).

Question (2)

An undamped oscillator with mass M and resonant frequency Ω_o is sitting quiescently at the origin $x = 0$. Then, at time $t = 0$, it is given a "kick", by a force F, which starts it moving. Subsequently, at a later time $t = t_1$, the previously undamped oscillator has a frictional damping switched on, adding a term $-\eta(dx/dt)$ to the equation of motion for the oscillator. This frictional damping remains on for all subsequent times.

Using any method you please (Fourier or Laplace transforms, or Green function methods, or any other method), solve for the motion of the oscillator for all times.