NEWTONIAN MECHANICS & GRAVITATION

Newton formulated what is now called 'classical mechanics'. Since his time the theory has been reformulated and generalized in various ways. These reformulations have made its basic assumptions a lot clearer, but without changing the essential basis of the theory. In what follows we first look at these essential features of classical mechanics, in a way which is a little different from that of Newton, summarizing the logical structure of the theory and its relationship to the real world. Then we look at Newton's original formulation, which is of great interest if we wish to understand the development of physics before and after Newton, and the philosophical developments which accompanied this. Newton's formulation was not just a mathematical theory, but a revision of basic philosophical concepts which were important at that time.

(1) BASIC STRUCTURE of CLASSICAL MECHANICS

This part of this section looks first at the the logical structure of the Newtonian framework, seen from a modern standpoint. Then we go on to understand in a more general way the concept of force, first looking at the universal gravitational force introduced by Newton, and then taking a more general look at forces, both from a Newtonian standpoint and from a more modern perspective.

LOGICAL STRUCTURE of NEWTONIAN MECHANICS

Newton formulated mechanics in terms of 3 laws of motion, which we start with here (note that in modern physics one does not formulate classical Newtonian physics in this way at all, but instead uses something called the 'principle of least action'- but this is not relevant to the present discussion). We can state these laws very quickly, without all the frills added by Newton, as follows:

1st Law: *A body continues in a state of uniform motion unless acted on by a force*

2nd Law: *A body of mass m acted on by a force $\mathbf{F}$ accelerates with an acceleration $\mathbf{a}$ such that $\mathbf{F} = m\mathbf{a}$*

3rd Law: *A force $\mathbf{F}$ acting on a body will be accompanied somewhere by an equal and opposite reactive force $-\mathbf{F}$.*

Three remarks are in order immediately, viz. (i) the first law is redundant- it is merely a special case of the 2nd law, with $\mathbf{a}$ and $\mathbf{F}$ taken to be zero; (ii) the mass m in the second law is referred to as the *inertial mass*, in deference to Newton, even though we now have a different view of inertia; (iii) in the case of the 3rd law, one allows explicitly for the possibility of 'non'local' interactions, ie., that a force $\mathbf{F}$ acts on one body at one place, with the reactive force $-\mathbf{F}$ acting on another body at another place. In this case it is natural to say that there is an *interaction* between them, acting over the pace between the 2 bodies. On the other hand one can also have contact interactions, acting between bodies that touch.

At first glance these laws seem relatively straightforward. A massive body moves and accelerates in response to forces, which can come from interaction with other bodies, either through short-range interactions (ie., when they touch) or long-range interactions such as gravity. Thus any change in motion of a body must come from interaction with another body, which must also change its motion in an equal and opposite way.

However a little thought shows that whereas it is easy how to relate the concept of acceleration, used in the 2nd law, to operations in the real world, this is not so obvious for the other quantities introduced (Force and mass). The modern way of looking at this would go something like the following:

(1) There are only 2 laws, the 2nd and the 3rd; the 1st is a special case of the 2nd.

(2) These laws are usefully thought of as 'axioms' (indeed this is what Newton called them). This means that apart from the definitions given of the terms used in the laws/axioms, any meaning assigned to the quantities mentioned has to arise from the axioms themselves, and from whatever connection is hypothesized between the quantities in the axioms, and phenomena in the real physical world.

(3) In the 2nd law 3 quantities are mentioned, viz., acceleration $\mathbf{a}$, force $\mathbf{F}$ (both vectors, ie., with magnitude and direction), and mass m (a scalar, ie., having only magnitude). In the 3rd law Force is again mentioned. How are these defined? To answer this we have to say what meaning is assigned to them by the axioms/laws, and also how they are related to physical quantities or operations. One answer to this question can be expressed as follows:

(i) The acceleration is simply defined mathematically as the rate of change of velocity, and the velocity is the rate of change of position (mathematically one says that $\mathbf{a} = d\mathbf{v}/dt$, where t is elapsed time, and the velocity $\mathbf{v} = d\mathbf{r}/dt$

where $\mathbf{r}$ is the position of the system concerned; hence $\mathbf{a} = d^2\mathbf{r}/dt^2$). The important point here is that acceleration is defined unambiguously in terms of time and space intervals. Since in the real world these are both defined by some kind of unambiguous measurement operation, we can say precisely what acceleration $\mathbf{a}$ means in the real world. (NB: The definition of space intervals used to be done by a standard bar in Paris, which defined a metre- now it is done in a more complex way using lasers. The measure of time is now given by atomic clocks).

(ii) Neither force nor mass are defined outside the framework of Newton's laws. Thus we must look to the laws themselves to define what these quantities mean. In the case of mass we proceed as follows- we note on the basis of the 3rd law and 2nd laws together that if 2 bodies interact, then the RATIO of their masses can be found by looking at how the interaction changes their velocities. From the 3rd law, the suffer equal and opposite forces. From the second law, the change in velocities caused by these forces is then going to be inversely proportional to their masses- this, if one mass is twice as big as the other, this bigger mass will suffer half the acceleration of the other (in algebra- since $\mathbf{F}_1 = -\mathbf{F}_2$, then $m_1\mathbf{a}_1 = -m_2\mathbf{a}_2$, so that $\mathbf{a}_1/\mathbf{a}_2 = -m_2/m_1$), and thus half the change in velocity. But this means that if we define one particular body as a *standard mass*(and call it., eg., 1 kilogram), then we can define all other masses in terms of this, by the simple operation of letting them interact with the first mass.

(iii) Now, what is the definition of force? This is more complicated. In fact we can say nothing more about what force is than what is contained in the laws, *until* more is said about the physical nature of these. This seems bizarre- what was apparently an intuitive concept has been reduced to an abstraction with no meaning apart from the laws- but this it turns out is how one makes progress. To see how this happens we must now turn to the example that Newton was able to analyse, ie., the gravitational force.

The GRAVITATIONAL FORCE

Newton was well aware that there were more than just gravitational forces in Nature (this is discussed below). However the only force he could give a precise understanding of was the gravitational force. Again, I give here a modern view of this force, and then later in this section the viewpoint of Newton is discussed. Newton's theory of gravitation can be summed up in two simple statements, viz;

(i) Any body with mass M is the source of a gravitational force, which is proportional to M. We call M the *gravitational mass* of the body, but according to the theory, this mass is EQUAL to the inertial mass defined by the 2nd law. This equivalence between the gravitational mass which is the *source* of gravitational force, and the inertial mass which *feels* it (according to the 2nd law) was later called the "*Principle of Equivalence*" by Einstein, and used to great effect by him.

(ii) Another mass m at a distance r from the first mass feels the force from M with a strength which is proportional to its own mass m (and again we assume the principle of equivalence), and also proportional to the inverse square of their separation, ie., proportional to $1/r^2$. This force is always attractive, along the line between them. In the same way the first mass M feels the force from the second one, and again it is attractive and along the line between them.

From these two statements we see that the force of one on the other is proportional to the *product* of their masses, and proportional to $1/r^2$. Thus the masses feel equal and opposite forces, directed towards each other, with a magnitude proportional to Mm/r^2. The universal law of gravitation is then written as

$$F = G\frac{Mm}{r^2} \tag{0.1}$$

where G is just some constant which determines the strength of the force, and F is the force attracting the two masses towards each other.

This is essentially what Newton postulated. Notice that all quantities on the right hand side of this equation have already been defined, except for the constant G, which has to be measured. Thus we have an unambiguous way of saying what the gravitational force is, quantitatively, once G has been measured. The theory can easily be tested- once G has been found by measuring the attractive force for one set of masses, it should be the same for any other pair of masses.

How to measure G? This is instructive. What we have to do is take 2 known masses, and then try to measure the acceleration each feels towards the other, caused by the gravitational force between them. The problem is that this force is REALLY SMALL, for any reasonable sized masses. Until the 19th century such an experiment was impossible in the laboratory, and so although one could verify that the celestial bodies certainly obeyed Newton's laws, and that the gravitational force obeyed the universal gravitation equation (in the sense that the force did fall off like $1/r^2$, and was proportional to Mm), one could not find out G for planets, since the actual values of the celestial masses was not known. Now in 21st century we have measured G, and so by simply observing the motions of planets, stars, etc., we can simply calculate their masses by using the formula above. In a later section on relativity I give more details on the measurement of G.

This concludes the discussion of the structure of Newton's mechanics. We see that nowadays we assume a fairly simple structure, with no reference to absolute space and time, or to any complex philosophical questions, or to the mechanisms leading to the gravitational force. It is not that these questions are meaningless- it is just that the structure of Newton's laws, and their validity, can be discussed without having to deal with the underlying question of structure. This is the real advance of Newton's work- his laws are valid no matter what the underlying cause of the forces. What these causes were would have to wait a very long time for clarification.

(2) NEWTON'S FORMULATION of MECHANICS

Newton's formulation of mechanics was much more complicated than the one just given. This, by the way, is almost always what happens- the initial formulation of a new theory is often very complex, and is progressively simplified (and often generalised) by later workers, who strip away what is unnecessary, or refine it to a clearer from. However there were very important philosophical issues at stake for Newton, and it is important to see what these were. I do this by introducing some of his ideas, and then, in 2 Appendices, letting him speak for himself by quoting at length from his writings.

NEWTONIAN DYNAMICS

Let us first go back to the formulation of Classical dynamics that was given in its original form by Newton. This was done in a series of 'Axioms', the 3 laws, which I repeat here verbatim:

Law I: *Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon*

Law II: *The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed*

Law III: *To every action there is always opposed an equal reaction; or the mutual actions of 2 bodies upon each other are always equal, and directed to contrary parts*

The meaning of these laws for Newton was similar to, but not the same as, the present meaning. The principal difference comes from Newton's understanding of the 1st Law, which we can glean from the following definition of the inherent or 'innate' force of matter (definition III of the preliminary definitions to the *Principia*):

Definition III: *The* 'vis insita', *or innate force of matter, is a power of resisting, by which every body, as much as it lies, endeavours to persevere in its present state, whether it be at rest, or of moving uniformly forward in a right line*

and Newton then adds by way of clarification that '*a body, from the inactivity of matter, is not without difficulty put out of its state of rest or motion. Upon which account, this* vis insita, *may, by a most significant name, be called* vis inertiae, *or force of inactivity. But a body exerts this force only when another force, impressed upon it, endeavours to change its condition...*"

To a modern reader this is very puzzling- as noted above, the 1st law seems to be just a special case of the 2nd law, applying when the force F in the 2nd law is taken to be zero. But this was not the case for Newton. This comes out most clearly in his discussion of absolute space and time- for Newton there was a unique rest frame, defined by absolute space, and so all motion was motion with respect to this frame. In order to make Newton's position on this clear, his 'Scholium' on the subject is reprinted in full at the end of this section.

Why Newton felt that the idea of absolute space had to be kept is not completely clear. He seems to have felt a deep antipathy to the idea that all uniform motion was merely relative (the modern view), even though there is nothing in his laws that requires absolute motion. It is also likely that he was reacting against Descartes's ideas about relative motion- he acquired a deep distaste for Cartesian philosophy at some point during his studies, for reasons that are not known. Finally, once Newton had plumped for the idea of gravitational force acting at a distance through a vacuum, it became important to find some other 'substrate' to replace the aether. The idea of 'action at a distance, with a force acting through a vacuum, was a real problem for Newton, in the absence of the aether as a substrate for the force. Nevertheless it was the conclusion to which he was driven by experiments.

The basic problem with absolute space and time is that they apparently make no difference to Newton's framework. One way to see this is to ask whether there is any way that one could test for the existence of absolute space- if there is no way to do this, then it is not clearly meaningful to talk about it. Newton was aware of this and did attempt to devise what we would now call a 'thought experiment' which would accomplish this. This thought experiment is usually known as 'Newton's rotating bucket experiment', and is also found in the Scholium in the Appendix to this section. The idea is that one can tell whether the bucket is rotating even in the absence of any visible frame of reference, simply by watching the shape of the water surface in the bucket. Newton then argues that it is rotating with respect to absolute space.

From the modern viewpoint, Newton simply made a mistake here. What the rotating bucket experiment shows is that the rotating water is being accelerated- the rotational motion necessarily involves acceleration since the water is changing direction its motion (actually it is just acceleration under a centripetal force). Thus it does not demonstrate uniform motion with respect to an absolute reference frame. At best it is an argument that rotational acceleration can be defined with respect to an absolute 'non-rotating' reference frame; but it gives no evidence that the bucket is not moving at some constant velocity with respect to this frame. To put it differently- the fact that acceleration may be absolute gives no reason to believe that either velocities or positions are absolute. However in the 19th and 20th centuries, physicists were still returning to the rotating bucket thought experiment, in an attempt to clarify Mach's principle- the idea that a non-rotating frame of reference is determined by the matter distribution in the universe. We will discuss this in a later section.

Moving now to the second law and third laws, we see that, just as now, the main questions that must be clarified concern the definition of force and mass. We have seen above how this can be done in a modern framework. However Newton looked at things in a somewhat different way. His idea of mass was the same as ours, but he was more reticent on the subject of force. Although he appreciated very well what it meant to treat force in a more abstract way than previous workers- simply in order go beyond a simply mechanical philosophy- nevertheless Newton worried constantly over how to find a deeper understanding of forces. In the *Principia* he hid these doubts as far as possible, and refrained from speculation:

'*our purpose is only to trace out the quantity and properties of this force from the phenomena, and to apply what we discover in some simple cases as principles, by which, in a mathematical way, we may estimate the effects thereof in more involved cases...in a mathematical way, to avoid all questions about the nature or quality of this force, which we would not be understood to determine by any hypothesis..*'

In the same way Newton was not prepared to speculate in print about what might be the underlying cause of the gravitational force, the one force that whose basic laws he had unravelled. In the final pages of the *Principia* he wrote:

'*Hitherto we have explained the phenomena of the heavens and of our sea by the power of gravity, but we have not yet assigned the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centres of the sun and the planets, without suffering the least diminution of its force; that operates not according to the quantity of the surfaces of the particles upon which it acts (as mechanical causes used to do), but according to the quantity of the solid matter which they contain, and propagates its virtue to immense distances, decreasing always in the duplicate proportion of the distances...but hitherto I have not been able to discover the cause of these properties of gravity from phaenomena, and I frame no hypotheses; for whatever is not deduced from the phaenomena is to be called a hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phaenomena, and afterwards rendered general by induction....And to us it is enough that gravity really does exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.*'

This passage is curious in the clear reference it also makes to the 'experimental philosophy' of Francis Bacon- there is no question that Newton was a firm believer in the empirical approach to science. Nevertheless in his unpublished writings Newton was indeed prepared to frame hypotheses, and speculate on just how far one could push his ideas, down to the microscopic realm. Writing in the 'it Conclusio' (his unpublished philosophical conclusion to the *Principia*), on all the unobservable phenomena and motions that might exist at this microscopic level, he says:

'*If anyone should have the good fortune to discover all of these, I might almost say that he will have laid bare the whole nature of bodies so far as the mechanical causes of things are concerned... Whatever reasoning holds for greater motions, should hold for lesser ones as well. The former depend on the greater attractive forces of larger bodies, and I suspect that the latter depend on the lesser forces, as yet unobserved, of insensible particles. For, from the forces of gravity, of magnetism, and of electricity it is manifest that there are various kinds of natural forces, and that there may be still more kinds is not to be rashly denied. It is very well known that greater bodies act mutually on each other by those forces, and I do not clearly see why lesser ones should not act on one another by similar forces.*'

This last passage is in some ways quite astonishing in its accurate prediction of how the subject would evolve. Just as remarkable is the final paragraph of the *Principia*, in which he says:

'*And now we might add something concerning a certain most subtle Spirit which pervades and lies hid in all gross bodies; by the force and action of which Spirit the particles of bodies mutually attract one another at near distances,* *as well repelling as attracting the neighbouring corpuscles; and light is emitted, reflected, refracted, inflected, and heats bodies; and all sensation is excited, and the members of animal bodies move at the command of the will, namely, by the vibrations of this Spirit, mutually propagated along the solid filaments of the nerves, from the outward organs of sense to the brain, and from the brain into the muscles. But these are things that cannot be explained in a few words, nor are we furnished with that sufficiency of experiments which is required for an accurate determination and demonstration of the laws by which this electric and elastic Spirit operates.*'

This valedictory passage of the *Principia* gives a hint of just how far Newton may have been framing his hypotheses- as far as making a rough guess at the existence of what, 2 centuries later, would be called the electromagnetic field.

APPENDIX: ABSOLUTE SPACE and TIME

As mentioned above, Newton felt that it was necessary to maintain a kind of 'ghost of the aether', in the from of absolute spatial and temporal frames. It seems easy to dismiss this idea, and it seemed to have been finally eliminated by Einstein's principle of relativity at the beginning of the 20th century. However it is worth paying attention to the arguments- particularly as the idea of the aether has re-emerged in a rather different form, in the vacuum of modern quantum field theory. Newton explained himself quite fully in his introduction to the *Principia*, which is reprinted here:

SCHOLIUM, from INTRODUCTION to the PRINCIPIA

Hitherto I have laid down the definitions of such words as are less known, and explained the sense in which I would have them to be understood in the following discourse. I do not define time, space, place, and motion, as being well known to all. Only I must observe, that the common people conceive those quantities under no other notions but from the relation they bear to sensible objects. And thence arise certain prejudices, for the removing of which it will be convenient to distinguish them into absolute and relative, true and apparent, mathematical and common.

I. Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year.

II. Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our sense determine by its position to bodies; and which is commonly taken for immovable space; such is the dimension of a subterraneous, an aerial, or celestial space, determined by its position in respect of the earth. Absolute and relative space are the same in figure and magnitude; but they do not remain always numerically the same. For if the earth, for instance, moves, a space of our air, which relatively and in respect of the earth remains always the same, will at one time be one part of the absolute space into which the air passes; at another time it will be another part of the same, and so, absolutely understood, it will be continually changed.

III. Place is a part of space which a body takes up, and is according to the space, either absolute or relative. I say, a part of space; not the situation, nor the external surface of the body. For the places of equal solids are always equal but their surfaces, by reason of their dissimilar figures, are often unequal. Positions properly have no quantity, nor are they so much the places themselves, as the properties of places. The motion of the whole is the same with the sum of the motions of the parts; that is, the translation of the whole, out of its place, is the same thing with the sum of the translations of the parts out of their places; and therefore the place of the whole is the same as the sum of the places as the parts, and for that reason, it is internal, and in the whole body.

IV. Absolute motion is the translation of a body from one absolute place into another; and relative motion, the translation from one relative place into another. Thus in a ship under sail, the relative place of a body is that part of the ship which the body possesses; or that part of the cavity which the body fills, and which therefore moves together with the ship: and relative rest is the continuance of the body in the same part of the ship, or of its cavity. But real, absolute rest, is the continuance of the body in the same part of that immovable space, in which the ship itself, its cavity, and all that it contains, is moved. Wherefore, if the earth is really at rest, the body, which relatively rests in the ship, will really and absolutely move with the same velocity which the ship has on the earth. But if the earth also moves, the true and absolute motion of the body will arise, partly from the true motion of the earth, in immovable space, partly from the relative motion of the ship on the earth; and if the body moves also relatively in the ship, its true motion will arise, partly from the true motion of the earth, in immovable space, and partly from the relative motions as well of the ship on the earth, as of the body in the ship; and from these relative motions will arise the relative motion of the body on the earth. As if that part of the earth, where the ship is, was truly moved towards the east, with a velocity of 10010 parts; which the ship itself, with a fresh gale, and full sails, is carried towards the west, with a velocity expressed by 10 of those parts; but a sailor walks in the ship towards the east, with 1 part of the said velocity; then the sailor will be moved truly in immovable space towards the east, with a velocity of 10001 parts, and relatively on the earth towards the west, with a velocity of 9 of those parts.

Absolute time, in astronomy, is distinguished from relative, by the equation or correction of the apparent time. For the natural days are truly unequal, though they are commonly considered as equal, and used for a measure of time; astronomers correct this inequality that they may measure the celestial motions by a more accurate time. It may be, that there is no such thing as an equable motion, whereby time may be accurately measured. All motions may be accelerated and retarded, but the flowing of absolute time is not liable to any change. The duration or perseverance of the existence of things remains the same, whether the motions are swift or slow, or none at all: and therefore this duration ought to be distinguished from what are only sensible measures thereof; and from which we deduce it, by means of the astronomical equation. The necessity of this equation, for determining the times of a phenomenon, is evinced as well from the experiments of the pendulum clock, as by eclipses of the satellites of Jupiter.

As the order of the parts of time is immutable, so also is the order of the parts of space. Suppose those parts to be moved out of their places, and they will be moved (if the expression may be allowed) out of themselves. For times and spaces are, as it were, the places as well of themselves as of all other things. All things are placed in time as to order of succession; and in space as to order of situation. It is from their essence or nature that they are places; and that the primary places of things should be movable, is absurd. These are therefore the absolute places; and translations out of those places, are the only absolute motions.

But because the parts of space cannot be seen, or distinguished from one another by our senses, therefore in their stead we use sensible measures of them. For from the positions and distances of things from any body considered as immovable, we define all places; and then with respect to such places, we estimate all motions, considering bodies as transferred from some of those places into others. And so, instead of absolute places and motions, we use relative ones; and that without any inconvenience in common affairs; but in philosophical disquisitions, we ought to abstract from our senses, and consider things themselves, distinct from what are only sensible measures of them. For it may be that there is no body really at rest, to which the places and motions of others may be referred.

But we may distinguish rest and motion, absolute and relative, one from the other by their properties, causes, and effects. It is a property of rest, that bodies really at rest do rest in respect to one another. And therefore as it is possible, that in the remote regions of the fixed stars, or perhaps far beyond them, there may be some body absolutely at rest; but impossible to know, from the position of bodies to one another in our regions, whether any of these do keep the same position to that remote body; it follows that absolute rest cannot be determined from the position of bodies in our regions.

It is a property of motion, that the parts, which retain given positions to their wholes, do partake of the motions of those wholes. For all the parts of revolving bodies endeavor to recede from the axis of motion; and the impetus of bodies moving forwards arises from the joint impetus of all the parts. Therefore, if surrounding bodies are moved, those that are relatively at rest within them will partake of their motion. Upon which account, the true and absolute motion of a body cannot be determined by the translation of it from those which only seem to rest; for the external bodies ought not only to appear at rest, but to be really at rest. For otherwise, all included bodies besides their translation from near the surrounding ones, partake likewise of their true motions; and though that translation were not made, they would not be really at rest, but only seem to be so. For the surrounding bodies stand in the like relation to the surrounded as the exterior part of a whole does to the interior, or as the shell does to the kernel; but if the shell moves, the kernel will also move, as being part of the whole, without removal from near the shell.

A property, near akin to the preceding, is this, that if a place is moved, whatever is placed therein moves along with it; and therefore a body, which is moved from a place in motion, partakes also of the motion of its place. Upon which account, all motions, from places in motion, are no other than parts of entire and absolute motions; and every entire motion is composed of the motion of the body out of its first place, and the motion of this place out its place; and so on, until we come to some immovable place, as in the before-mentioned example of the sailor. Wherefore, entire and absolute motions can be no otherwise determined than by immovable places; and for that reason I did before refer those absolute motions to immovable places, but relative one to movable places. Now no other places are immovable but those that, from infinity to infinity, do all retain the same given position one to another; and upon this account must ever remain unmoved; and do thereby constitute immovable space.

The causes by which true and relative motions are distinguished, one from the other, are the forces impressed upon bodies to generate motion. True motion is neither generated nor altered, but by some force impressed upon the body moved; but relative motion may be generated or altered without any force impressed upon the body. For it is sufficient only to impress some force on other bodies with which the former is compared, that by their giving way, that relation may be changed, in which the relative rest or motion of this other body did consist. Again, true motion suffers always some change from any force impressed upon the moving body; but relative motion does not necessarily undergo any change by such forces. For if the same forces are likewise impressed on those other bodies, with which the comparison is made, that the relative position may be preserved, then that condition will be preserved in which the relative motion consists. And therefore any relative motion may be changed when the true motion remains unaltered, and the relative may be preserved when the true suffers some change. Thus, true motion by no means consists in such relations.

The effects which distinguish absolute from relative motion are, the forces of receding from the axis of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute circular motion, they are greater or less, according to the quantity of the motion. If a vessel, hung by a long cord, is so often turned about that the cord is strongly twisted, then filled with water, and held at rest together with the water; thereupon, by the sudden action of another force, it is whirled about the contrary way, and while the cord is untwisting itself, the vessel continues for some time in this motion; the surface of the water will at first be plain, as before the vessel began to move; but after that, the vessel, by gradually communicating its motion to the water, will make it begin sensibly to revolve, and recede by little and little from the middle, and ascent to the sides of the vessel, forming itself into a concave figure (as I have experienced), and the swifter the motion becomes, the higher will the water rise, till at last, performing its revolutions in the same times with the vessel, it becomes relatively at rest in it. This ascent of the water shows its endeavor to recede from the axis of its motion; and the true and absolute circular motion of the water, which is here directly contrary to the relative, becomes known, and may be measured by this endeavor. At first, when the relative motion of the water in the vessel was greatest, it produced no endeavor to recede from the axis; the water showed no tendency to the circumference, nor any ascent towards the sides of the vessel, but remained on a plain surface, and therefore its true circular motion had not yet begun. But afterwards, when the relative motion of the water had decreased, the ascent thereof towards the sides of the vessel proved its endeavor to recede from the axis; and this endeavor showed the real circular motion of the water continually increasing, till it had acquired its greatest quantity, when the water rested relatively in the vessel. And therefore this endeavor does not depend upon any translation of the water in respect of the ambient bodies, nor can true circular motion be defined by such translation. There is only one real circular motion of any one revolving body, corresponding to only one power of endeavoring to recede from its axis of motion, as its proper and adequate effect; but relative motions, in one and the same body, are innumerable, according to the various relations it bears to external bodies, and like other relations, are altogether destitute of any real effect, any otherwise than they may perhaps partake of that one only true motion. And therefore in their system who suppose that our heavens, revolving below the sphere of the fixed stars, carry the planets along with them; the several parts of those heavens, and the planets, which are indeed relatively at rest in their heavens, do yet really move. For they change their position one to another (which never happens to bodies truly at rest), and being carried together with their heavens, partake of their motions, and as parts of revolving wholes, endeavor to recede from the axis of their motions.

Wherefore relative quantities are not the quantities themselves, whose names they bear, but those sensible measures of them (either accurate or inaccurate), which are commonly used instead of the measured quantities themselves. And if the meaning of words is to be determined by their use, then by the names time, space, place, and motion, their sensible measures are properly to be understood; and the expression will be unusual, and purely mathematical, if the measured quantities themselves are meant. On this account, those violate the accuracy of language, which ought to be kept precise, who interpret these words for the measured quantities. Nor do those less defile the purity of mathematical and philosophical truths, who confound real quantities with their relations and sensible measures.

It is indeed a matter of great difficulty to discover, and effectually to distinguish, the true motions of particular bodies from the apparent; because the parts of that immovable space, in which those motions are performed, do by no means come under the observation of our senses. Yet the thing is not altogether desperate; for we have some arguments to guide us, partly from the apparent motions, which are the differences of the true motions; partly from the forces, which are the causes and effects of the true motions. For instance, if two globes, kept at a given distance one from the other by means of a cord that connects them, were revolved about their common centre of gravity, we might, from the tension of the cord, discover the endeavor of the globes to recede from the axis of their motion, and from thence we might compute the quantity of their circular motions. And then if any equal forces should be impressed at once on the alternate faces of the globes to augment or diminish their circular motions, from the increase or decrease of the tension of the cord, we might infer the increment or decrement of their motions; and thence would be found on what faces those forces ought to be impressed, that the motions of the globes might be most augmented; that is, we might discover their hindmost faces, or those which, in the circular motion, do follow. But the faces which follow being known, and consequently the opposite ones that precede, we should likewise know the determination of their motions. And thus we might find both the quantity and the determination of this circular motion, even in an immense vacuum, where there was nothing external or sensible with which the globes could be compared. But now, if in that space some remote bodies were placed that kept always a given position one to another, as the fixed stars do in our regions, we could not indeed determine from the relative translation of the globes among those bodies, whether the motion did belong to the globes or the bodies. But if we observed the cord, and found that its tension was that very tension which the motions of the globes required, we might conclude the motion to be in the globes, and the bodies to be at rest; and then, lastly, from the translation of the globes among the bodies, we should find the determination of their motions. But how we are to obtain the true motions from their causes, effects, and apparent differences, and the converse, shall be explained more at large in the following treatise. For to this end it was that I composed it.

EXTRACTS from LEIBNIZ-CLARKE CORRESPONDENCE

The debate between G. W. Leibniz (1646-1716) and Samuel Clarke (1675-1729) was essentially a debate between Leibniz and Newton, with Clarke acting as Newton's proxy. The debate touches on the same issue discussed already, that of absolute space and time. As Leibniz argued, if space is uniform , there is no particular reason to single out any particular frame of reference as being fundamental. Space is basically just a set of relations between material bodies. This is of course the view that was later employed by Einstein in his framing of the principle of relativity- it is also the view that had been previously adopted by Galileo. Here I just give a few extracts from the debate:

LEIBNIZ:
The great foundation of mathematics is the principle of contradiction, or identity, that is, that a proposition cannot be true and false at the same time; and that therefore A is A, and cannot be not A. This single principle is sufficient to demonstrate every part of arithmetic and geometry, that is, all mathematical principles. But in order to proceed from mathematics to natural philosophy, another principle is requisite, as I have observed in my Theodicy: I mean, the principle of a sufficient reason, viz. that nothing happens without a reason why it should be so, rather than otherwise. And therefore Archimedes being to proceed from mathematics to natural philosophy, in his book De Aequilibrio, was obliged to make use of a particular case of the great principle of a sufficient reason. He takes it for granted, that if there be a balance, in which everything is alike on both sides, and if equal weighted are hung on the two ends of that balance, the whole will be at rest. It is because no reason can be given, why one side should weigh down, rather than the other. Now, by that single principle, viz. that there ought to be a sufficient reason why things should be so, and not otherwise, one may demonstrate the being of God, and all the other parts of metaphysics or natural theology; and even, in some measure, those principles of natural philosophy, that are independent upon mathematics: I mean, the dynamical principles, or the principles of force. (Leibniz's 2nd paper)

CLARKE:
It is very true, that nothing is, without a sufficient reason why it is, and why it is thus rather than otherwise. And therefore, where there is no cause, there can be no effect. But this sufficient reason is oft-times no other, than the mere will of God. For instance: why this particular system of matter, should be created in one particular place, and that in another particular place; when, (all place being absolutely indifferent to all matter,) it would have been exactly the same thing vice versa, supposing the two systems (or the particles) of matter to be alike; there can be no other reason, but the mere will of God. Which if it could in no case act without a predetermining cause, any more than a balance can move without a preponderating weight; this would tend to take away all power of choosing, and to introduce fatality. (Clarke's 2nd Reply)

LEIBNIZ:
5. ... I say then, that if space was an absolute being, there would something happen for which it would be impossible there should be a sufficient reason. Which is against my axiom. And I prove it thus. Space is something absolutely uniform; and, without the things placed in it, one point of space does not absolutely differ in any respect whatsoever from another point of space. Now from hence it follows, (supposing space to be something in itself, besides the order of bodies among themselves,) that it is impossible there should be a reason, why God, preserving the same situations of bodies among themselves, should have placed them in space after one certain particular manner, and not otherwise; why every thing was not placed the quite contrary way, for instance, by changing East into West. But if space is

nothing else, but that order or relation; and is nothing at all without bodies, but the possibility of placing them; then those two states, the one such as it now is, the other supposed to be the quite contrary way, would not at all differ from one another. Their difference therefore is only to be found in our chimerical supposition of the reality of space in itself. But in truth the one would exactly be the same thing as the other, they being absolutely indiscernible; and consequently there is no room to enquire after a reason of the preference of the one to the other.

6. The case is the same with respect to time. Supposing any one should ask, why God did not create every thing a year sooner; and the same person should infer from thence, that God has done something, concerning which it is not possible there should be a reason, why he did it so, and not otherwise: the answer is, that his inference would be right, if time was any thing distinct from things existing in time. For it would be impossible there should be any reason, why things should be applied to such particular instants, rather than to others, their succession continuing the same. But then the same argument proves, that instants, consider'd without the things, are nothing at all; and that they consist only in the successive order of things; which order remaining the same, one of the two states, viz. that of a supposed anticipation would not at all differ, nor could be discerned from, the other which now is. (Leibniz's 3rd Paper)

7. It appears from what I have said, that my axiom has not been well understood; and that the author denies it, tho' he seems to grant it. It is true, says he, that there is nothing without a sufficient reason why it is, and why it is thus, rather than otherwise: but he adds, that this sufficient reason, is often the simple or mere will of God: as, when it is asked why matter was not placed elsewhere in space; the same situations of bodies among themselves being preserved. But this is plainly maintaining, that God wills something, without any sufficient reason for his will: against the axiom, or the general rule of whatever happens. This is falling back into the loose indifference, which I have confuted at large, and showed to be absolutely chimerical even in creatures, and contrary to the wisdom of God, as if he could operate without acting by reason. As for my own opinion, I have said more than once, that I hold space to be something merely relative, as time is; that I hold it to be an order of coexistences, as time is an order of successions. For space denotes, in terms of possibility, an order of things which exist at the same time, considered as existing together; without enquiring into their manner of existing. And when many things are seen together, one perceives that order of things among themselves (Leibniz's 3rd Paper).

CLARKE:

2. Undoubtedly nothing is, without a sufficient reason why it is, rather than not; and why it is thus, rather than otherwise. But in things in their own nature indifferent; mere will, without any thing external to influence it, is alone that sufficient reason. As in the instance of God's creating or placing any particle of matter in one place rather than in another, when all places are originally alike. And the case is the same, even though space were nothing real, but only the mere order of bodies: for still it would be absolutely indifferent, and there could be no other reason but mere will, why three equal particles should be placed or ranged in the order a, b, c, rather than in the contrary order. And therefore no argument can be drawn from this indifference of all places, to prove that no space is real. For different spaces are really different or distinct one from another, though they be perfectly alike. And there is this evident absurdity in supposing space not to be real, but to be merely the order of bodies; that, according to that notion, if the earth and sun and moon had been placed where the remotest fixed stars now are, (provided they were placed in the same order and distance they now are with regard one to another,) it would not only have been, (as this learned author rightly says,) la meme chose, the same thing in effect; which is very true: but it would also follow, that they would then have been in the same place too, as they are now: which is an express contradiction.

4. If space was nothing but the order of things coexisting; it would follow, that if God should remove in a straight line the whole material world entire, with any swiftness whatsoever; yet it would still always continue in the same place: and that nothing would receive any shock upon the most sudden stopping of that motion. And if time was nothing but the order of succession of created things; it would follow, that if God had created the world millions of ages sooner than he did, yet it would not have been created at all the sooner. Further: space and time are quantities; which situation and order are not.

5. The argument in this paragraph, is; that because space is uniform or alike, and one part does not differ from another; therefore the bodies created in one place, if they had been created in another place, (supposing them to keep the same situation with regard to each other,) would still have been created in the same place as before: which is a manifest contradiction. The uniformity of space, does indeed prove, that there could be no (external) reason why God should create things in one place rather than in another: but does that hinder his own will, from being to itself a sufficient reason of acting in any place, when all places are indifferent or alike, and there be good reason to act in some place? (Clarke's 3rd Reply)

LEIBNIZ:

5. Those great principles of a sufficient reason, and of the identity of indiscernibles, change the state of metaphysics. That science becomes real and demonstrative by means of these principles; whereas before, it did generally consist in

empty words.

6. To suppose two things indiscernible, is to suppose the same thing under two names. And therefore to suppose that the universe could have had at first another position of time and place, than that which it actually had; and yet that all the parts of the universe should have had the same situation among themselves, as that which they actually had; such a supposition, I say, is an impossible fiction.

15. It is a like fiction, (that is) an impossible one, to suppose that God might have created the world some millions of years sooner. They who run into such kind of fictions, can give no answer to one that should argue for the eternity of the world. For since God does nothing without reason, and no reason can be given why he did not create the world sooner; it will follow, either that he has created nothing at all, or that he created the world before any assignable time, that is, that the world is eternal. But when once it has been shown, that the beginning, whenever it was, is always the same thing; the question, why it was not otherwise ordered, becomes needless and insignificant.

16. If space and time were any thing absolute, that is, if they were any thing else, besides certain orders of things; then indeed my assertion would be a contradiction. But since it is not so, the hypothesis [that space and time are any thing absolute] is contradictory, that is, it is an impossible fiction.

17. And the case is the same as in geometry; where by the very supposition that a figure is greater than it really is, we sometimes prove that it is not greater. This indeed is a contradiction; but it lies in the hypothesis, which appears to be false for that very reason. (Leibniz's 4th Paper)

CLARKE:

5 and 6. Two things, by being exactly alike, do not cease to be two. The parts of time, are as exactly like to each other, as those of space: yet two points of time, are not the same point of time, nor are they two names of only the same point of time. Had God created the world but this moment, it would not have been created at the time it was created. And if God has made (or can make) matter finite in dimensions, the material universe must consequently be in its nature moveable; for nothing that is finite, is immoveable. To say therefore that God could not have altered the time or place of the existence of matter, is making matter to be necessarily infinite and eternal, and reducing all things to necessity and fate.

13. If the world be finite in dimensions, it is moveable by the power of God and therefore my argument drawn from that mobility is conclusive. Two places, though exactly alike, are not the same place. Nor is the motion or rest of the universe, the same state; any more than the motion or rest of a ship, is the same state, because a man shut up in the cabin cannot perceive whether the ship sails or not, so long as it moves uniformly. The motion of the ship, though the man perceives it not, is a real different state, and has real different effects; and, upon a sudden stop, it would have other real effects; and so likewise would an indiscernible motion of the universe. To this argument, no answer has ever been given. It is largely insisted on by Sir Isaac Newton in his Mathematical Principles, (Definition 8.) where, from the consideration of the properties, causes, and effects of motion he shows the difference between real motion, or a body's being carried from one part of space to another; and relative motion, which is merely a change of the order or situation of bodies with respect to each other. This argument is a mathematical one; showing, from real effects, that there may be real motion where there is none relative; and relative motion, where there is none real: and is not to be answered, by barely asserting the contrary.

14. The reality of space is not a supposition, but is proved by the foregoing arguments, to which no answer has been given. Nor is any answer given to that other argument, that space and time are quantities, which situation and order are not.

15. It was no impossibility for God to make the world sooner or later than he did: nor is it at all impossible for him to destroy it sooner or later than it shall actually be destroyed. As to the notion of the world's eternity; they who suppose matter and space to be the same, must indeed suppose the world to be not only infinite and eternal, but necessarily so: even as necessarily as space and duration, which depend not on the will, but on the existence of God. But they who believe that God created matter in what quantity, and at what particular time, and in what particular spaces he pleased, are here under no difficulty. For the wisdom of God may have very good reasons for creating this world, at that particular time he did; and may have made other kinds of things before this material world began, and may make other kinds of things after this world is destroyed.

16 and 17. That space and time are not the mere order of things, but real quantities (which order and situation are not;) has been proved above (See Third Reply, No. 4) and no answer yet given to those proofs. And till an answer be given to those proofs, this learned author's assertion is (by his own confession in this place) a contradiction. (Clarke's 4th Reply)

EXTACTS from LEIBNIZ's 5th PAPER:

27. The parts of time or place, considered in themselves, are ideal things: and therefore they perfectly resemble

one another like two abstract units. But it is not so with two concrete ones, or with two real times, or two spaces filled up, that is, truly actual.

28. I don't say that two points of space are one and the same point, nor that two instants of time are one and the same instant, as the author seems to charge me with saying. But a man may fancy, for want of knowledge, that there are two different instants, where there is but one: in like manner as I observed in the 17th paragraph of the foregoing answer, that frequently in geometry we suppose two, in order to represent the error of a gainsayer, when there is really but one. If any man should suppose that a right line cuts another in two points; it will be found after all, that those two pretended points must coincide, and make but one point.

29. I have demonstrated, that space is nothing else but an order of the existence of things, observed as existing together; and therefore the fiction of a material finite universe, moving forward in an infinite empty space, cannot be admitted. It is altogether unreasonable and impracticable. For, besides that there is no real space out of the material universe; such an action would be without any design in it: it would be working without doing any thing, agendo nihil agere. There would happen no change, which could be observed by any person whatsoever. These are imaginations of philosophers who have incomplete notions, who make space an absolute reality. Mere mathematicians, who are only taken up with the conceits of imagination, are apt to forge such notions; but they are destroyed by superior reasons.

31. I don't grant, that every finite is moveable. According to the hypothesis of my adversaries themselves, a part of space, though finite, is not moveable. What is moveable, must be capable of changing its situation with respect to something else, and to be in a new state discernible from the first: otherwise the change is but a fiction. A moveable finite, must therefore make part of another finite, that any change may happen which can be observed.

33. Since space in itself is an ideal thing, like time; space out of the world must needs be imaginary, as the schoolmen themselves have acknowledged. The case is the same with empty space within the world; which I take also to be imaginary, for the reason before alleged.

34. The author objects against me the vacuum discovered by Nr. Guerike of Madenburg, which is made by pumping the air out of a receiver; and he pretends that there is truly a perfect vacuum, or a space without matter, (at least in part,) in that receiver. The Aristotelians and Cartesians, who do not admit a true vacuum, have said in answer to that experiment of Mr. Guerike, as well as to that of Torricelli of Florence, (who emptied the air out of a glass-tube by the help of quicksilver,) that there is no vacuum at all in the tube or in the receiver; since glass has small pores, which the beams of light, the effluvia of the load-stone, and other very thin fluids may go through. I am of their opinion.

47. I will here show, how men come to form to themselves the notion of space. They consider that many things exist at once and they observe in them a certain order of co-existence, according to which the relation of one thing to another is more or less simple. This order, is their situation or distance. When it happens that one of those co-existent things changes its relation to a multitude of others, which do not change their relation among themselves; and that another thing, newly come, acquires the same relation to the others, as the former had; we then say, it is come into the place of the former; and this change, we call a motion in that body, where in is the immediate cause of the change. And though many, or even all the co-existent things, should change according to certain known rules of direction and swiftness; yet one may always determine the relation of situation, which every co-existent acquires with respect every other co-existent; and even that relation which any other co-existent would have to this, or which this would have to any other, if it had not changed, or if it had changed any otherwise. And supposing or feigning, that among those coexistents, there is a sufficient number of them, which have undergone no change; then we may say, that those which have such a relation to those fixed existents, as others had to them before, have now the same place which those others had. And that which comprehends all those places, is called space. Which shows, that in order to have an idea of place, and consequently of space, it is sufficient to consider these relations, and the rules of their changes, without needing to fancy any absolute reality out of the things whose situation we consider. And, to give a kind of a definition: place is that, which we say is the same to A and, to B, when the relation of the co-existence of B, with C, E, F, G etc. agrees perfectly with the relation of the co-existence, which A had with the same C, E, F, G, etc. It may be said also, without entering into any further particularity, that place is that, which is the same in different moments to different existent things, when their relations of co-existence with certain other existents, which are supposed to continue fixed from one of those moments to the other, agree entirely together. And fixed existents are those, in which there has been no cause of any change of the order of their co-existence with others; or (which is the same thing,) in which there has been no motion. Lastly, space is that, which results from places taken together.

And here it may not be amiss to consider the difference between place, and the relation of situation, which is in the body that fills up the place. For, the place of A and B, is the same; whereas the relation of A to fixed bodies, is not precisely and individually the same, as the relation which B (that comes into its place) will have to the same fixed bodies; but these relations agree only. For, two different subjects, as A and B, cannot have precisely the same individual affection; it being impossible, that the same individual accident should be in two subjects, or pass from one subject to another. But the mind not contented with an agreement, looks for an identity, for something that should be truly the same and conceives it as being extrinsic to the subjects: and this is what we call place and space. But this can only be an ideal thing; containing a certain order, wherein the mind conceives the application of relations.

In like manner, as the mind can fancy to itself an order made up of genealogical lines, whose bigness would consist only in the number of generations, wherein every person would have his place: and if to this one should add the fiction of a metempsychosis, and being in the same human souls again; the persons in those lines might change place; he who was a father, or a grandfather, might become a son, or a grandson, etc. And yet those genealogical places, lines, and spaces, though they should express real truth, would only be ideal things.

I shall allege another example, to show how the mind uses, upon occasion of accidents which are in subjects, to fancy to itself something answerable to those accidents, out of the subjects. The ratio or proportion between two lines L and M, may be conceived three several ways; as a ratio of the greater L, to the lesser M; as a ratio of the lesser M, to the greater L; and lastly, as something abstracted from both, that is, as the ratio between L and M, without considering which is the antecedent, or which the consequent; which the subject, and which the object. And thus it is, that proportions are considered in music. In the first way of considering them, L the greater; in the second, M the lesser, is the subject of that accident, which philosophers call relation. But, which of them will be the subject, in the third way of considering them? It cannot be said that both of them, L and M together, are the subject of such an accident; for if so, we should have an accident in two subjects, with one leg in one, and the other in the other; which is contrary to the notion of accidents. Therefore we must say, that this relation, in this third way of considering it, is indeed out of the subjects; but being neither a substance, nor an accident, it must be a mere ideal thing, the consideration of which is nevertheless useful.

To conclude: I have here done much like Euclid, who not being able to make his readers well understand what ratio is absolutely in the sense of geometricians; defines what are the same ratios. Thus, in like manner, in order to explain what place is, I have been content to define what is the same place. Lastly; I observe, that the traces of moveable bodies, which they leave sometimes upon the immoveable ones on which they are moved; have given men occasion to form in their imagination such an idea, as if some trace did still remain, even when there is nothing unmoved. But this is a mere ideal thing, and imports only, that if there was any unmoved thing there, the trace might be marked out upon it. And it is this analogy, which makes men fancy places, traces and spaces; though those things consist only in the truth of relations, and not at all in any absolute reality.

52. In order to prove that space, without bodies, is an absolute reality; the author objected, that a finite material universe might move forward in space. I answered, it does not appear reasonable that the material universe should be finite; and, though we should suppose it to be finite; yet it is unreasonable it should have motion any otherwise, than as its parts change their situation among themselves; because such a motion would produce no change that could be observed, and would be without design. It is another thing, when its parts change their situation among themselves; for then there is a motion in space; but it consists in the order of relations which are changed. The author replies now, that the reality of motion does not depend upon being observed; and that a ship may go forward, and yet a man, who is in the ship may not perceive it. I answer, motion does not indeed depend upon being observed; but it does depend upon being possible to be observed. There is no motion, when there is no change that can be observed. And when there is no change that can be observed, there is no change at all. The contrary opinion is grounded upon the supposition of a real absolute space, which I have demonstratively confuted by the principle of the want of a sufficient reason of things.

53. I find nothing in the Eighth Definition of the Mathematical Principles of Nature, nor in the Scholium belonging to it, that proved, or can prove, the reality of space in itself. However, I grant there is a difference between an absolute true motion of a body, and a mere relative change of its situation with respect to another body. For when the immediate cause of the change is in the body, that body is truly in motion; and then the situation of other bodies, with respect to it, will be changed consequently, though the cause of the change be not in them. It is true that, exactly speaking, there is not any one body, that is perfectly and entirely at rest; but we frame an abstract notion of rest, by considering the thing mathematically. Thus have I left nothing unanswered, of what has been alleged for the absolute reality of space. And I have demonstrated the falsehood of that reality, by a fundamental principle, one of the most certain both in reason and experience; against which, no exception or instance can be alleged. Upon the whole, one may judge from what has been said that I ought not to admit a moveable universe; nor any place out of the material universe.

54. I am not sensible of any objection, but what I think I have sufficiently answered. As for the objection that space and time are quantities, or rather things endowed with quantity; and that situation and order are not so: I answer, that order also has its quantity; there is in it, that which goes before, and that which follows; there is distance or interval. Relative things have their quantity, as well as absolute ones. For instance, ratios or proportions in mathematics, have their quantity, and are measured by logarithms; and yet they are relations. And therefore though

time and space consist in relations, yet they have their quantity.

55. As to the question, whether God could have created the world sooner; it is necessary here to understand each other rightly. Since I have demonstrated, that time, without things, is nothing else but a mere ideal possibility; it is manifest, if any one should say that this same world, which has been actually created, might have been created sooner, without any other change; he would say nothing that is intelligible. For there is no mark or difference, whereby it would be possible to know, that this world was created sooner. And therefore, (as I have already said,) to suppose that God created the same world sooner, is supposing a chimerical thing. It is making time a thing absolute, independent upon God; whereas time does only co-exist with creatures, and is only conceived by the order and quantity of their changes.

56. But yet, absolutely speaking, one may conceive that an universe began sooner, than it actually did. Let us suppose our universe, or any other, to be represented by the Figure AF; and that the ordinate AB represents its first state; and the ordinates CD, EF, its following states: I say, one may conceive that such a world began sooner, by conceiving the figure prolonged backwards, and by adding to it SRABS. For thus, things being increased, time will be also increased. But whether such an augmentation be reasonable and agreeable to God's wisdom, is another question, to which we answer in the negative; otherwise God would have made such an augmentation. ... The case is the same with respect to the destruction of the universe. As one might conceive something added to the beginning, so one might also conceive something taken off towards the end. But such a retrenching from it, would be also unreasonable.

57. Thus it appears how we are to understand, that God created things at what time he pleased; for this depends upon the things which he resolved to create. But things being once resolved upon, together with their relations; there remains no longer any choice about the time and the place, which of themselves have nothing in them real, nothing that can distinguish them, nothing that is at all discernible.

58. One cannot therefore say, as the author does here, that the wisdom of God may have good reasons to create this world at such or such a particular time: that particular time, considered without the things, being an impossible fiction; and good reasons for a choice, being not to be found, where everything is indiscernible.

62. I don't say that matter and space are the same thing. I only say, there is no space, where there is no matter; and that space in itself is not an absolute reality. Space and matter differ, as time and motion. However, these things, though different, are inseparable.

67. The parts of space are not determined and distinguished, but by the things which are in it: and the diversity of things in space, determines God to act differently upon different parts of space. But space without things, has nothing whereby it may be distinguished; and indeed not any thing actual.

68. If God is resolved to place a certain cube of matter at all, he is also resolved in what particular place to put it. But it is with respect to other parts of matter; and not with respect to bare space itself, in which there is nothing to distinguish it.

104. I don't say, that space is an order or situation, which makes things capable of being situated: this would be nonsense. Any one needs only consider my own words, and add them to what I said above, (Numb. 47) in order to show how the mind comes to form to itself an idea of space, and yet that there need not be any real and absolute being answering to that idea, distinct from the mind, and from all relations. I don't say therefore, that space is an order or situation, but an order of situations; or (an order) according to which, situations are disposed; and that abstract space is that order of situations, when they are conceived as being possible. Space is therefore something [merely] ideal. But, it seems, the author will not understand me. I have already, in this paper, (Numb. 54,) answered the objection, that order is not capable of quantity.

105. The author objects here, that time cannot be an order of successive things, because the quantity of time may become greater or less, and yet the order of successions continue the same. I answer; this is not so. For if the time is greater, there will be more successive and like states interposed; and if it be less, there will be fewer; seeing there is no vacuum, nor condensation, or penetration, (if I may so speak), in times, any more than in places.