PHYS 306: HOMEWORK ASSIGNMENT No. 7: RIGID BODY ROTATION

(March 23rd, 2017)

HOMEWORK DUE: Friday, MARCH 31st, 2017
To be handed in during class- Late Homework will not be accepted

QUESTION (1) PERTURBATIONS and the EULER EQUATIONS: Suppose we have a solid body with moments of inertia I_1, I_2, I_3 along its principal axis directions $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$, and it is rotating with angular velocity $\underline{\boldsymbol{\omega}}(t)$ having components $(\omega_1, \omega_2, \omega_3)$ along these axes.

1(a) Let's suppose that the $I_1 > I_2 > I_3$, but that I_1 and I_2 are very similar, so that $(I_1 - I_2) \ll (I_2 - I_3)$; in this way we have a small dimensionless parameter in the problem. Write the solutions in the form $\omega_j(t) = \Omega_j(t) + \sum_{k=1} \eta_j^{(k)}(t)\, \epsilon^k$, where ϵ is the small parameter and where $j = 1, 2, 3$; and thence derive the Euler equations in the form of an expansion in terms of this parameter. Find, to lowest approximation, the solution for the dynamics of $\underline{\boldsymbol{\omega}}(t)$, assuming that at $t = 0$, we have that $\omega_1(t = 0) = \bar{\Omega}_1^{(0)}$, that $\omega_2(t = 0) = 0$ and that $\omega_3(t = 0) = \Omega_3^{(0)}$; here $\bar{\Omega}_1^{(0)}$ and $\Omega_3^{(0)}$ are constants.

2(b) Now let us go to the next lowest order in your small parameter. Write out the equations of motion for each of the corrections to the solutions you found above for the $\omega_j(t)$; and then solve the equation of motion for the correction to $\omega_3(t)$ up to this next order. Add this to the solution you already found in (a) to get the complete solution for $\omega_3(t)$.

2(c) Find the magnitude and direction of the angular momentum $\mathbf{L}$, given these initial conditions, and to the lowest approximation you've used above; in (i) the principal axis frame of the body, and (ii) in an inertial frame (assume here that at $t = 0$, the inertial frame axes coincide with the principal axis frame).

END of 7th HOMEWORK ASSIGNMENT

NB: This will be the last homework assignment for this course